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Controlled estimation algorithms of disparity map using a compensation compression scheme for stereoscopic image coding

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Dedication

*Dedicated to
My Mother and ,
My Husband,
and in memory of
My Father.*



Abstract

Nowadays, 3D technology is of ever growing demand because stereoscopic imaging create an immersion sensation. However, the price of this realistic representation is the doubling of information needed for storage or transmission purpose compared to 2D image because a stereoscopic pair results from the generation of two views of the same scene. This thesis focused on stereoscopic image coding and in particular improving the disparity map estimation when using the Disparity Compensated Compression (DCC) scheme.

Classically, when using Block Matching algorithm with the DCC, a disparity map is estimated between the left image and the right one. A predicted image is then computed. The difference between the original right view and its prediction is called the residual error. This latter, after encoding and decoding, is injected to reconstruct the right view by compensation (i.e. refinement) . Our first developed algorithm takes into account this refinement to estimate the disparity map. This gives a proof of concept showing that selecting disparity according to the compensated image instead of the predicted one is more efficient. But this done at the expense of an increased numerical complexity. To deal with this shortcoming, a simplified modelling of how the JPEG coder, exploiting the quantization of the DCT components, used for the residual error yields with the compensation is proposed. In the last part, to select the disparity map minimizing a joint bitrate-distortion metric is proposed. It is based on the bitrate needed for encoding the disparity map and the distortion of the predicted view. This is by combining two existing stereoscopic image coding algorithms.

Keywords : Stereoscopic Image Coding, Block Matching Algorithm, Disparity Map Estimation, Disparity Compensation, Quantization, JPEG, Rate-Distortion



Résumé

Ces dernières années ont vu apparaître de nombreuses applications utilisant la technologie 3D tels que les écrans auto-stéréoscopiques ou encore la visio-conférence stéréoscopique. Cependant ces applications nécessitent des techniques bien adaptées pour comprimer efficacement le volume important de données à transmettre ou à stocker. Les travaux développés dans cette thèse concernent le codage d'images stéréoscopiques et s'intéressent en particulier à l'amélioration de l'estimation de la carte de disparité dans un schéma de Compression avec Compensation de Disparité (CCD). Habituellement, l'algorithme d'appariement de blocs similaires dans les deux vues permet d'estimer la carte de disparité en cherchant à minimiser l'erreur quadratique moyenne entre la vue originale et sa version reconstruite sans compensation de disparité. L'erreur de reconstruction est ensuite codée puis décodée afin d'affiner (compenser) la vue prédite. Pour améliorer la qualité de la vue reconstruite, dans un schéma de codage par CCD, nous avons prouvé que le concept de sélectionner la disparité en fonction de l'image compensée plutôt que de l'image prédite donne de meilleurs résultats. En effet, les simulations montrent que notre algorithme non seulement réduit la redondance inter-vue mais également améliore la qualité de la vue reconstruite et compensée par rapport à la méthode habituelle de codage avec compensation de disparité. Cependant, cet algorithme de codage requiert une grande complexité de calculs. Pour remédier à ce problème, une modélisation simplifiée de la manière dont le codeur JPEG (à savoir la quantification des composantes DCT) impacte la qualité de l'information codée est proposée. En effet, cette modélisation a permis non seulement de réduire la complexité de calculs mais également d'améliorer la qualité de l'image stéréoscopique décodée dans un contexte CCD. Dans la dernière partie, une métrique minimisant conjointement la distorsion et le débit binaire est proposée pour estimer la carte de disparité en combinant deux algorithmes de codage d'images stéréoscopiques dans un schéma CCD.

Mots clés : Codage d'image stéréoscopique, Appariement de blocs, Estimation de carte de disparité, Compensation de disparité, Quantification, JPEG, Débit-distorsion

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Notations and Abbreviations

Notations

I_l	Original left view of the stereoscopic pair
I_r	Original right view of the stereoscopic pair
I_p	Predicted right view without compensation
\hat{I}_r	Compensated right view
\hat{I}_l	Reconstructed left view
R	Residual image
\hat{R}	Reconstructed residual image
C_{ql}	Lossy encoding operation of the left image
q_l	Parameter controlling C_{ql}
Q_l	Set of all allowed q_l
D_l	Decoding operation of the left view
C_{qr}	Lossy encoding operation of the residual image
q_r	Parameter controlling C_{qr}
Q_r	Set of all allowed q_r
D_r	Decoding operation of the residual image
d	Disparity map
C	Lossless encoding operation of the disparity map
D	Decoding operation of the disparity map
s	Disparity value
S	Searching window
b	Bitrate
b_a	Target bitrate
k	Block number
K	Total block number
(i_k, j_k)	Block position in the block coordinates system
(i, j)	Block position in the pixel coordinates system
B	A bloc
(Δ_i, Δ_j)	Displacements in the bloc B
J	Cost function
d_k	Disparity value of block k

α	Scaling factor
T	Orthogonal matrix
$\rho(q_r)$	Average distortion ratio (function of q_r)

Abbreviations

PSNR	Peak Signal to Noise Ratio
DC	Disparity Compensation
DCC	Disparity Compensated Compression scheme
DE	Disparity Estimation
IP	Image Prediction
BM	Block Matching
DCBM	Disparity Compensated Block Matching
FDCBM	Fast Disparity Compensated Block Matching
COMB	Combining the R and FDCBM algorithms
DCT	Discrete Cosine Transform
IDCT	Inverse Discrete Cosine Transform
2D	Two dimensional
3D	Three dimensional
AVC	Advanced Video Coding
MVV	Multi View Video
MVC	Multi View Coding
Stereo	Stereoscopic



Introduction

1.1 Thesis context

DURING the last decade, the use of stereoscopic imaging technology has greatly increased. These images add the depth information which gives a more realistic perceptual sensations. Many applications uses stereo imaging such as 3D cinema, telemedecine, entertainment, cartography, remote sensing...

In fact, a stereoscopic pair is composed of two views (i.e. images) of the same realistic-world scene with slightly different angles. The brain merges these two images (i.e. left image and right image) to give the relief effect. This represents the fundamental idea of stereoscopic vision.

Since the stereoscopic pair represents the same scene, the size of information is doubled compared to the traditional 2D image. To deal with this issue, an efficient compression technique is required to rapidly transmit and compactly store such data. Similar to monoscopic coding technique, stereo image compression can exploit independently the redundancy on each view. But thus doesn't take advantage of the correlation between the two images as their content is similar. In fact a pixel in the 3D scene is projected onto two points respectively in the left and right views. The estimation of the distance between all homologous pixels in each view forms the disparity map. Hence to take advantage of the binocular dependency, a scheme based on the Disparity

Compensation (DC) is investigated and proposed in this thesis.

The principle of DC scheme is first to choose one of the two images as the reference image (say the left image for instance). A disparity map is then estimated to predict the target image (here the right image). After that a residual error is computed as the difference between the original right view and its prediction. Finally the encoding left image, the compressed residual and the lossless coding of the disparity map are sent to the decoder side.

Estimating disparity map is very crucial in stereoscopic image coding because it impacts the performance in terms of rate distortion. While the ground truth disparity map is used for 3D reconstruction, The work of this manuscript focuses on disparity estimation for stereoscopic image compression purpose. It is worth noting that the disparity map accomplishing good performance can be different from the true disparity map.

Note that this work was done in the context of a joint supervision contrat between the Université Sorbonne Paris Nord (USPN) and the National Ingeneeing School of Tunisia (ENIT). It is carried out between the L2TI (Laboratoire de Transport et Traitement de l'Information) and LSITI (Laboratoire de Signal, Images et Technologies de l'Information) laboratories.

1.2 Objectives and contributions

The objective of this PhD thesis concerns the improvement of stereoscopic image coding algorithms of the state of the art. We are specially interested in improving the Disparity Compensated Compression scheme. At the outline of our work is the idea that the estimation of the disparity map should take into account, the impact of the encoded residual error between the original image and its prediction to refine the predicted view, instead of assuming that the best predicted view yields the best compensated one as it is done in the state of the art.

Our main contributions are summarized as following:

While the classical Disparity Compensated Compression (DCC) scheme in most cases based on Block Matching (BM) algorithm, yields a disparity map with which the predicted image is most similar to its original version. Then, the residual error

computed between the original right view and its prediction version is encoded, yielding a refinement added to the predicted image. In this manuscript, we propose a novel approach which first, improves all the possible predicted images taking into account this refinement, and then, estimates the disparity map as the one with which the predicted image resembles most that same view. This strategy shows a proof-of-concept work that selecting disparities according to the compensated view instead of the predicted view improves the rate-distortion performance. But this is done at the expense of the significant increase in numerical complexity.

To overcome this drawback, an other contribution concerns the computation of the blockwise disparity using a simplified model of how the encoder used for the residual error deals with the compensation refinement. This computation is done with the derived analytic expression exploiting in what causes distortion in such coder, namely the quantization of the DCT components as in JPEG coder. Indeed, disparity selection is no longer based on the computation of the whole compensated view with all its pixel values. Only 8×8 pixel values of the residual error are taken into account. Thus numerical complexity is definitely reduced. Basically disparity related blocks are exactly the JPEG related blocks (8×8). Our algorithm can be extended to larger blocks.

The aim of our last contribution is to select the disparity map accomplishing the best trade off between the minimization of the bitrate allocated to the disparity map and the minimization of the distortion of the predicted view. For this, the proposed strategy consists in combining two existing algorithms. The first one, called R-algorithm [1]. It is more efficient at low bitrates The second one, is already described in the previous contribution. It performs better at high bitrates.

1.3 Organization of the thesis

This dissertation is organized as follows.

Chapter 2 describes first the basic notions of stereo vision specially the principle of stereoscopic vision, the geometrical models used by stereoscopic systems and the methods used to compute disparities. Then we give an overview of stereoscopic image coding.

Chapter 3 shows how finding the best performing disparity map can be regarded as solving an optimization problem. The classical BM algorithm is derived as a sub-optimal solution. Then we describe the proposed Disparity Compensated Block

Matching (DCBM) algorithm as an other sub-optimal solution which takes into account the compensation effect. Simulations results are presented to compare the developed algorithm with the the classical BM approach with compensation. Due to its good performance in terms of rate-distortion, this chapter is a good proof of the proposed concept which, to our knowledge, has not been proposed in the state of the art.

Chapter 4 is concerned with finding the optimal solution to the optimization problem to solve, in particular, the computational complexity of the developed algorithm introduced in the previous chapter. A fast version of the DCBM, called FDCBM algorithm, by approximating the distortion between blocks of the compensated image and its original version by blocks of decoded residual error and its original version is proposed. This is thanks to an adequate modelling of the JPEG coding. This algorithm is also extended to blocks larger than 8×8 pixels. Experiments show that our proposal performs better than the BM with compensation, DCBM algorithms and another algorithm of the state of the art.

Chapter 5 proposes to combine the developed FDCBM algorithm and an existing algorithm of the state of the art. The proposed algorithm called COMB, jointly optimizes the bitrate and the distortion.

Finally, Chapter 6 concludes this manuscript and gives some directions for further investigation.

1.4 Publications

- **International journal paper**

I. Kadri, G. Dauphin, A. Mokraoui and Z. Lachiri, Disparities selection controlled by the compensated image quality for a given bitrate, *Signal, Image and Video Processing*, February 2020, DOI 10.1007/s11760-020-01643-1.

- **International conference paper**

I. Kadri, G. Dauphin, A. Mokraoui and Z. Lachiri, Stereoscopic Image Coding Performance using Disparity-Compensated Block Matching Algorithm, *Signal Processing: Algorithms, Architectures, Arrangements and Applications (IEEE SPA conference)*, Poznan, Poland, 2019.

I. Kadri, G. Dauphin, A. Mokraoui and Z. Lachiri, Stereoscopic image coding using a global disparity estimation algorithm optimizing the compensation scheme impact, Signal Processing: Algorithms, Architectures, Arrangements and Applications (IEEE SPA conference), Poznan, Poland, 2020 (Submitted).

- **National conference paper**

I. Kadri, G. Dauphin, A. Mokraoui, Z. Lachiri, Algorithme d'appariement de blocs basé sur l'optimisation de la compensation de disparité pour le codage d'image stéréoscopique, Compression et Représentation des Signaux Audiovisuels (CORESA), 12-14 Novembre 2018, Poitiers.

State of the art on stereoscopic image coding

Summary

OVER the last decade, the use of stereo imaging technology has significantly increased. Applications concerns the entertainment, industry, medical fields and cartography. A stereo image is composed of two views (left and right) perceived as two viewpoints of the same scene. Consequently, the amount of information to be stored or transmitted is doubled compared to the monoscopic image. That's why it is important to efficiently encoding them.

This chapter outlines the basic notions to understand the stereoscopic imaging. Firstly, we introduce the fundamental aspects of stereo vision. Secondly, we present an overview of stereoscopic image coding.

2.1 Stereoscopic vision aspects

2.1.1 Related concepts

Stereopsis results from the fusion of two pictures presented to the brain from left and right eyes as shown in Fig. 2.1. Each eye looks at the real scene from a slightly different perspective. The merging of these two images gives a depth sense (3D perception).

To mimic the human visual system the easiest way is to put two cameras side by side with a side shifting for acquiring left and right images of the same scene. Fig. 2.2

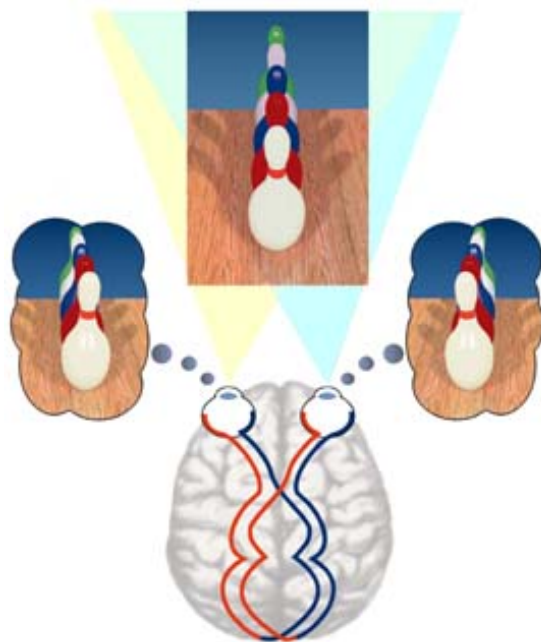


Figure 2.1: *Binocular vision* (source: [2]).

presents an example of stereo camera system.



Figure 2.2: *Stereoscopic camera* (source: [3]).

2.1.2 Stereoscopic geometry/systems of stereo acquisition

In order to describe image acquisition, the simplest model is the pinhole (linear projective) one. This model is illustrated in Fig. 2.3. It is composed by a retinal plane Ω_r where the image is formed, an optical center o placed at a focal distance f of the optical system. This system is contained in the focal plane Ω_f . The optical axis passes through the center of projection O . This line is perpendicular to Ω_r . In fact, it has demonstrated

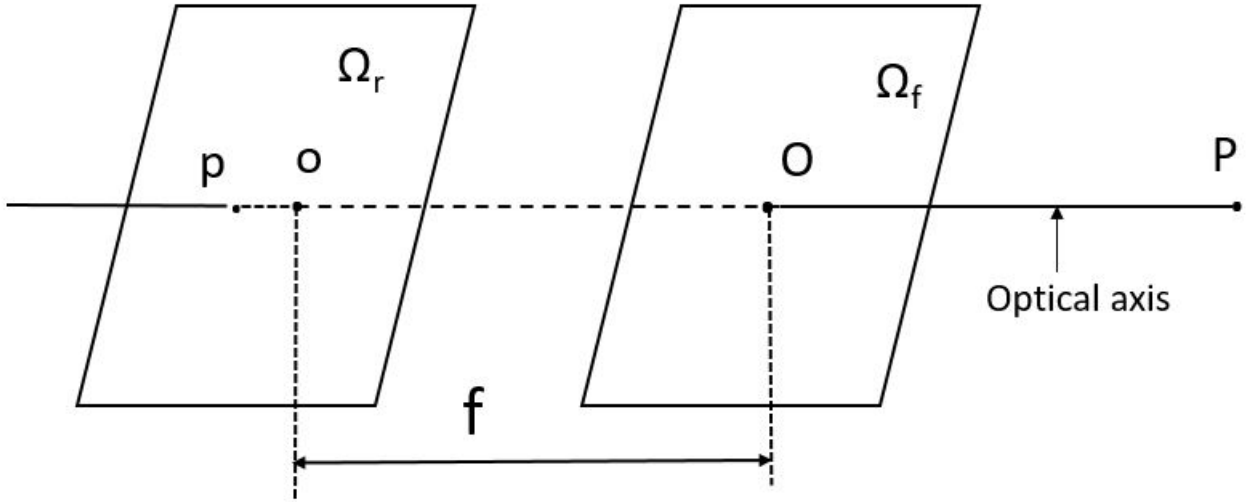


Figure 2.3: *Pinhole camera model* (source: [4]).

that the camera pinhole model can approximate efficiently the geometry and optics of most modern cameras [4]. It maps 3D point $P(X,Y,Z)$ to 2D point $p(x,y)$ as:

$$x = f \frac{X}{Z}, y = f \frac{Y}{Z} \quad (2.1)$$

where f is the focal distance

2.1.2.1 Epipolar constraint

Epipolar geometry describes the relationship between homologous points in left and right images. The different elements involved in this geometry are depicted in Fig . 2.4. The epipolar plan Ω contains optical centers of left and right camera respectively O_l and O_r , the 3D point P and its projection P_l and P_r . The baseline is the line joining the camera centers. The two points e_l and e_r result from the intersection of the baseline with the image plane are called epipoles. Let us now consider only the left projection P_l and we search for its correspondant point in the right image. The point P belongs necessarily to the line $O_l P_l$. The right projection P_r is located on the line l_r where the epipolar plane intersect the right image (due to the co-planarity). This line refers to the epipolar line associated to P_r (similarly for l_l the conjugate epipolar line). Not that all the epipolar lines in the image pass through the epipole point.

After explaining the geometric aspect of the epipolar constraint, we present the

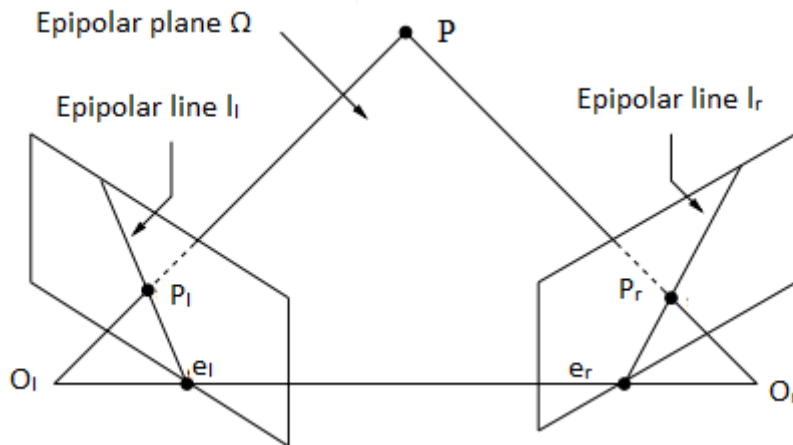


Figure 2.4: *Epipolar geometry (source: [5]).*

algebraic aspect, formalized by the fundamental matrix F as:

$$P_r F^T P_l = 0 \quad (2.2)$$

F is a 3×3 matrix depending on the intrinsic and extrinsic parameters of the cameras [6, 7]. By intrinsic parameters we mean the optical geometric, and digital characteristics of the camera and by extrinsic parameters orientations and positions of each camera. For detailed description of estimation techniques of the fundamental matrix, a complete review is cited in [7].

To recapitulate, the epipolar constraint overcome the complexity of searching homologous points. The search filed is restricted to 1D lines instead of the whole 2D image. This process is called rectification.

2.1.2.2 Rectification

In stereoscopic vision we distinguish mainly two camera dispositions. Fig. 2.4 illustrates the first configuration in which cameras are rotated towards each others by a small angle. This angle gives an inclination of epipolar lines. In the second camera disposition, the optical axes of cameras as well epipolar lines are parallel. Those latter coincide with the scanning lines as depicted in Fig. 2.5. This simplify the correspondence search. So homologous points have the same vertical coordinate. Indeed, rectification is the process that converts the first configuration to the simplified second one by involving transformation which projects the two images onto the same plane. A survey of the diverse existing rectification methods can be found in [8–10].

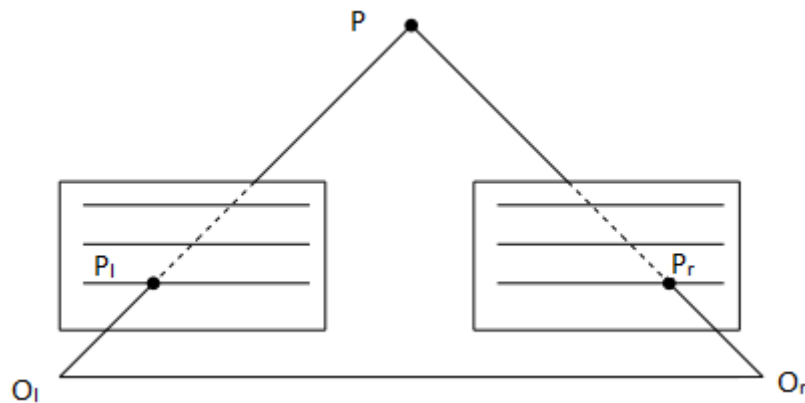


Figure 2.5: *Epipolar rectification (source: [5]).*

2.1.3 Methods of disparity estimation

Despite the reduction of the research field for correspondent points with epipolar geometry, it does not give any information about the exact location of both homologous points. Indeed, disparity is the difference in the x-coordinates of the corresponding pixels in the left and right views. Dense disparity map results from carrying out stereo matching for all pixels. After that, depth in 3D space is derived from this spatial shift. The literature divides disparity estimation methods mainly into two classes: local and global disparity estimation methods.

2.1.3.1 Local stereo correspondence methods

Local methods measure the similarity between two sets of pixels. They can be basically subdivided into two approaches: feature based approaches and area based ones. In the first category, high level feature like edges [11], curves [12] or segments [13] are compared. These features represent geometric properties of a scene which makes these methods accurate. Nevertheless, the step of feature extraction should be involved. Another disadvantage is related to the sparsity of the estimated disparity map which requires an interpolation step to complete disparities. This is not the case with area based algorithms which estimate directly a dense disparity map. This kind of methods is efficient in highly textured regions but sensitive to locally ambiguous zones due for example to occlusion and textureless contents. Area based methods correlate pixels or regions. That's why they are known as correlation-based approaches. Its principle is depicted in Fig. 2.5. Let us consider a pixel P_r in the right image. How can we find its homologous point in the left image? We start by selecting a window/bloc

surrounding P_r . After that, this bloc is moved in the left image search area to search a match window. At every shift a cost function is computed. Finally, the retained

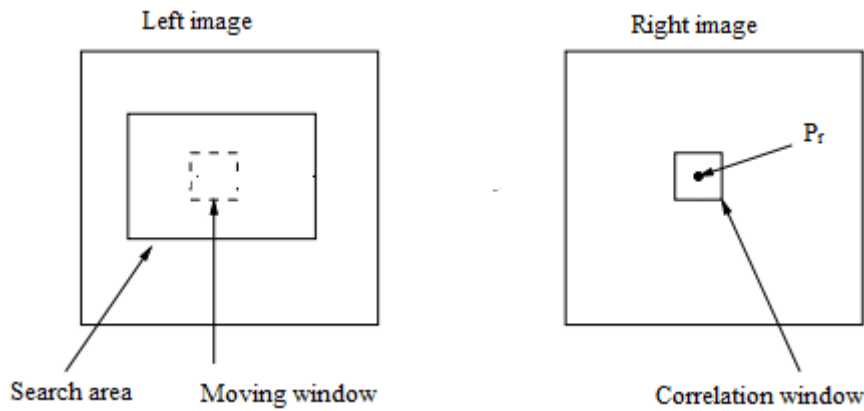


Figure 2.6: Correlation-based stereo matching approaches (source: [5]).

match is the one that gives the lowest cost. Many similarity measures can be used such as the Sum of Square Differences (SSD), the Sum of Absolute Differences (SAD) and the Normalized Cross-Correlation (NCC). The choice of window size and shape is the main inconvenient of correlation based methods. To overcome this limitation many approaches have been proposed which consider variable shape and size adaptively to the pixels intensity variation in the image [14–16].

2.1.3.2 Global stereo correspondence methods

Global methods aim at minimizing a global energy function $E(d)$ to estimate the disparity map. This latter defines the cost based on the measure of the displacement between correspondent pixels $E_{data}(d)$ and a regularization term $E_{smooth}(d)$ called smoothing prior which carries on the smoothness of the final disparity map. $E(d)$ takes the following form:

$$E(d) = E_{data}(d) + \alpha E_{smooth}(d) \quad (2.3)$$

where α is a positif term and d the estimated disparity map.

Dynamic programming (DP) [17, 18] is one of the most commonly used global methods. Its principle is as following: Let's take two scanlines (with the epipolar constraint) of size L . First a $L \times L$ cost matrix for all possible matching pixels is built. Then, best matches are selected taking into account the ordering and smoothness constraints. But

DP suffers from the well-known streaking effects which consist on the inconsistency problem between horizontal and vertical scanlines. To overcome this limitation a two pass dynamic programming has been proposed in [18].

To cope with the streaking artifacts, Roy and Cox [19] have proposed graph cuts methods. Its underlying idea is to transform the stereo problem in a pixel labelling one in order to construct a graph with minimal cut. Variational approaches [20] and belief propagation [21] are also used as techniques for global methods.

2.2 Reviewing the state of the art on stereoscopic image compression

Before presenting the overview on stereo image coding, let us give some background on image compression.

2.2.1 Basic concepts of image compression

The goal of compression is the reduction of the bits' number needed to represent an image. They are either lossless or lossy compression approaches. In lossless compression approaches, the decoded image is exactly the same as its original version. Contrary to the lossy compression approaches, the decoded image is an approximation of the original image. As depicted in Fig. 2.7, a typical framework of image compression includes basically three steps: transformation, quantization and entropy coding.

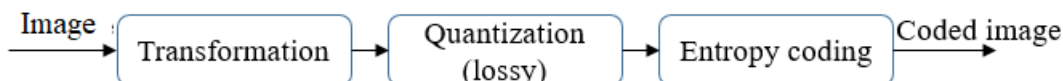


Figure 2.7: *General image compression framework.*

2.2.1.1 Transformation

In still image, neighboring pixels have redundant spatial information because they are correlated. Consequently, this redundancy has been exploited, for the purpose of efficient encoding, using a transformation step. Basically two approaches can be considered such as predictive coding and transformation one.

Predictive coding: Due to the high correlation between pixels, a better strategy is to encode the difference between neighboring instead of encoding each pixel separately.

The most common approach is Differential Pulse Code Modulation (DPCM) [22].

Transform coding: Redundancy in the transform domain is reduced greatly. So it's more efficient to code an image in its transformed version than in its original one. The Discrete Cosine Transform (DCT) [23] and the Discrete Wavelet Transform (DWT) [24] are the most used transforms in image and video compression.

The DCT transforms an image from its spatial representation to the frequency one. It has been used in different image/video coding standards such as JPEG (Joint Photographic Experts Group) [25] and MPEG coding standards (H263 [26], H264 [27], HEVC [28]) . The DCT, as a block transform, operates as following. The image is divided into non overlapping blocks for example 8×8 pixels and then the DCT is applied. The main drawback of DCT is the blocks' artifacts at low bitrates. On the other hand, DWT overcomes the drawbacks of the DCT. It is used in the JPEG 2000 standard [29]. Wavelet gives a representation in the spatial and scale domains of the image.

2.2.1.2 Quantization

Quantization is an irreversible operator (lossy). It converts a continuous signal into a discrete one. Two classes of quantization can be defined: vector and scalar. While vector quantization operates on blocks of transform coefficients, scalar quantization treats independently transform coefficients one by one.

It is worth pointing out that scalar quantization is less complex and is frequently used in practice. It can be used in two ways. Uniform quantization is suitable when the coefficients to be quantized are uniformly distributed. They are separated by a fixed distance called quantization step Q_{50} . Then the quantization is given by:

$$\forall x \in \mathbb{R} \quad f(x) = Q_{50} \times \left[\frac{x}{Q_{50}} \right] \quad (2.4)$$

where $[.]$ is the round operator and x (i.e. the coefficient) a real number. On the other hand, for non distributed coefficients, adaptive quantization should be performed. It is

the case for JPEG [25] which uses the following table for a quality factor equal to 50:

$$Q_{50} = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix} \quad (2.5)$$

We notice that this matrix uses on the top left smaller step quantization values compared to the remaining values. This is because the DCT transform properties which promotes low frequencies levels compared to high frequencies ones.

2.2.1.3 Entropy coding

After quantization, the resulting values must be encoded. The entropy coding transforms the quantized coefficients into a bitstream. Huffman coding and arithmetic coding are adopted in most image /video coding standards such as the JPEG standard [25]. Many other algorithms have been developed such as the Run Length Encoder(RLE)used in the JPEG2000 standard [29].

2.2.2 Stereoscopic image /video coding approaches

A stereoscopic video sequence is the result of two video sequence representing respectively the left and right camera. Because video sequences are taken at the same time with different perspectives, video coding take advantage of the redundancy in the temporal domain (inter frame correlation) and the spatial domain (intra frame correlation). Frames are characterized by disparities between views and motion between temporally successive captured images.

The Multiview video (MVV) is a general representation of multiple views. The Mutiview Video Coding (MVC) [30] standard, as an extension to the H.264/Advanced Video Coding (AVC), is the first standard dealing with two or more than two views. To code MVV, this standard exploits the interview correlation. Especially, images from different views can be used as references to predict current block. Note that the base view, chosen as the reference one, is encoded in simulcast H.264/AVC [5].To carry on the temporal and interview redundancies, motion and disparity compensation are used to encode non

base view [5] as described in the following section.

2.2.3 Disparity compensated scheme

Stereoscopic images present similar contents from slightly different perspective and the spatial displacements yield to the disparity map. In the purpose of exploiting inter-image redundancies more efficient coding schemes have been proposed based on the disparity compensation (DC) technique which is analog to motion compensation scheme used in video coding [31]. The DC process involves the main following steps:

- (i) One of the two views is taking as the reference one (for example the left one).
- (ii) A disparity map is then estimated between the left and right images.
- (iii) The target image (the right one) is then predicted based on the estimated disparity map.
- (iv) A residual image is computed as the difference between the original right view and its prediction.
- (v) This residual error is added to the predicted view in order to find the reconstructed target image by compensation.

The reference view and the residual error are encoded after a transformation coding. The disparity map is lossless encoded using an entropy coder. Finally, these information are sent to the decoder in order to recover the stereo pair.

Two stereoscopic coding schemes are proposed in the literature: Open Loop (OL) based structure and Closed Loop (CL) based structure. They are shown respectively in Fig. 2.8 and Fig. 2.9. They differ in the disparity estimation level.

As illustrated in Fig. 2.8, in the OL case, disparity map computation considers the original left view. This is not the case with the CL case which use the decoded left view as depicted in Fig. 2.9.

When decoding, whatever the structure, the left view is decoded at the beginning. The right view is then predicted based on the decoded left image and disparity map. To obtain the final reconstructed right view, the decoded residual error is added to the predicted right view (the compensation). When using the OL scheme, the reference view at the encoder and decoder are different which makes this structure sub-optimal.

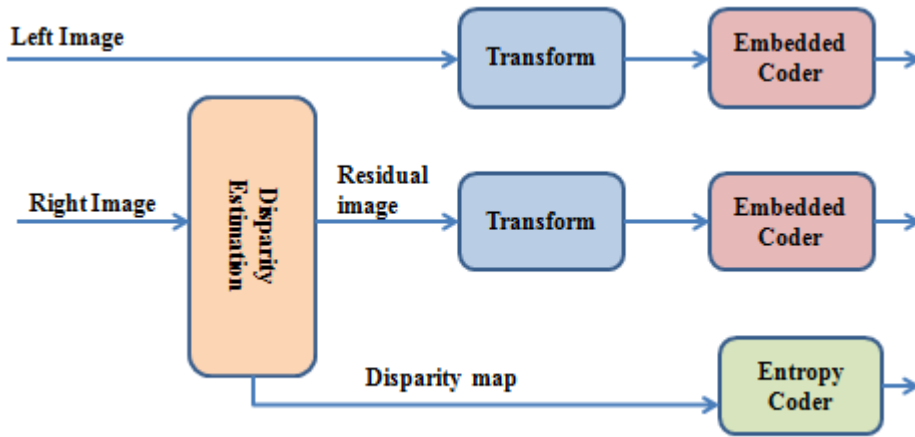


Figure 2.8: *Open Loop for stereoscopic image coding scheme [32].*

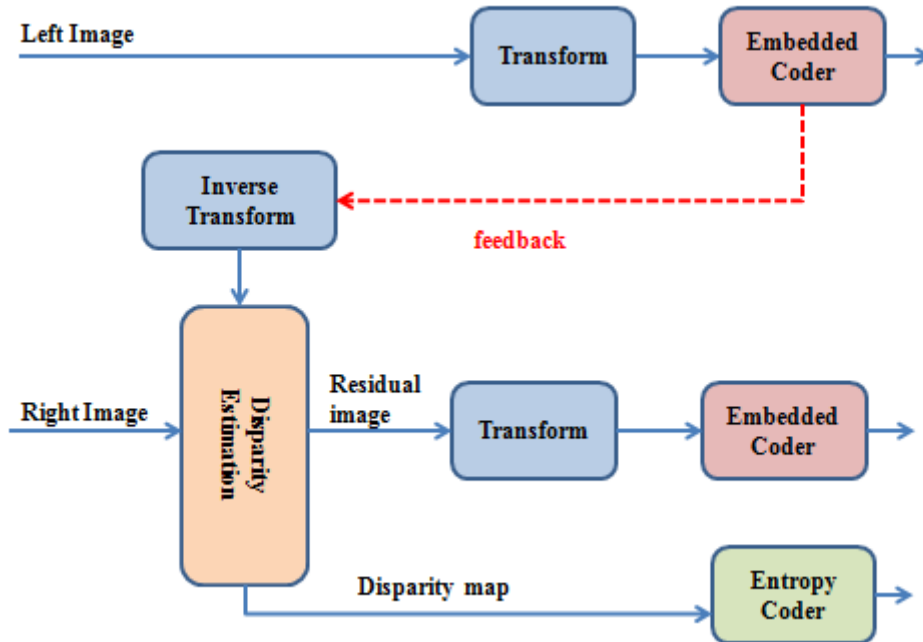


Figure 2.9: *Closed Loop for stereoscopic image coding scheme [32].*

To deal with this problem, the CL scheme use the decoded reference view at coding and decoding sides.

2.2.4 Disparity estimation for stereo image coding

The estimation of the disparity map is a crucial operation because it has an impact on the performance of the compressed stereoscopic image in terms of rate-distortion. Blockwise disparity is usually used because one disparity value is assigned to all pixels of the same block requiring therefore a few bit budget to encode the stereoscopic image. Indeed, several coding algorithms are based on the disparity compensated scheme. In

[33], disparity map is sequentially estimated minimizing a joint entropy distortion metric modelling the compromise between the minimization of the distortion of the predicted image (before compensation) and the minimization of the entropy needed to encode the disparity map. This is achieved by building a tree where each path represents a partial disparity map. This proposal extends only the M best paths, at each depth, by all possible disparities. This algorithm is also extended for non rectified stereoscopic image in [34]. The proposal in [35] is an extension of [33] which considers also blocks of variable sizes. In [1], Kadaikar and al. estimate also the disparity map using the same entropy distortion metric. Disparity map yielded by the BM algorithm is first taken as the reference one. This initial map is successfully modified when improvements are possible in terms of rate distortion.

Two approaches are proposed, in [36], for selecting disparities, from a large search window, minimizing the predicted image distortion for a given bitrate. The initial disparity map is the one computed by the BM algorithm. In [35], a new method for estimating disparity map using blocks of variable size is developed. This is by reducing the prediction view distortion and the bitrate needed to encode the disparity and the block length maps.

A region-based scheme for stereo image coding is presented in [37]. Basically occluded and non occluded regions are considered. Occluded ones are independently encoded. Whereas non occluded regions are segmented firstly into edge and smooth zones. Then they are divided in blocks of fixed size to be used in the block matching algorithm. Disparity map is losslessly encoded using residual uniform scalar quantizer followed by an adaptive arithmetic encoder.

Furthermore, disparity estimation and compensation are done after the DCT transformation of the right and left images in [38]. In [39] presents a wavelet-based stereo image coding scheme is proposed. Before disparity estimation and compensation a wavelet transform is applied to the stereo pair. To encode the wavelet coefficients a Subspace Projection Technique (SPT) is performed. Authors in [40] presented a method where disparity map and residual error are computed in the bandlet domain. In [41] a joint coding scheme using the Vector Lifting Scheme (VLS) to represent the stereoscopic pair instead of residual error is presented. The generated dense and smooth disparity map is then integrated in the VLS scheme.

Conclusion

This chapter presented the main concepts of stereoscopic imaging. A review of previous work on stereoscopic image coding approaches using the disparity compensated technique has been given. The following chapters will present our contributions on stereoscopic image coding.

Concept proof of the disparity map estimation controlled by the compensation error

Summary

The main objective of this chapter is to confirm the relevance of the new strategy of estimating the disparity map when the stereoscopic coding is based on a compensation scheme. The DC classical approach is based on Block Matching (BM) algorithm, yielding a disparity map with which the predicted image is most similar to its original version. Then, with no modification of the disparity map, the residual error is encoded, yielding a refinement added to the predicted image. Our proposal improves all the possible predicted images taking into account this refinement, and then, estimates the disparity map as the one with which the predicted image after compensation resembles most to its original version.

The remainder of this chapter is organized as follows. First, we present the optimization problem. Then we provide the description of the proposed algorithm. After that, experimental results are illustrated. Finally conclusions are drawn. This contribution resulted in an international and national conferences [42, 43].

3.1 Problem statement

3.1.1 Basic concepts and notations

In this chapter, the classical disparity compensated coding scheme with rectified stereoscopic images is considered.

The following notations are illustrated on Fig. 3.1 showing both the classical disparity coding and decoding schemes separated by a dashed line.

In Fig. 3.1, \mathbf{I}_l (upper left corner) denotes the left view chosen here as the reference view. It feeds a lossy encoder denoted C_{q_l} (upper left corner) where $q_l \in \mathbf{Q}_l$ is its quality factor and \mathbf{Q}_l is a set containing all allowed values. The bit stream output is transmitted to the decoder (left downward arrow connecting the dashed line). This bit stream is decoded by D_l yielding a reconstructed left view denoted $\widehat{\mathbf{I}}_l$ (lower left corner) as follows:

$$\widehat{\mathbf{I}}_l = D_l(C_{q_l}(\mathbf{I}_l)). \quad (3.1)$$

Note that the framework chosen uses a close loop as this bit stream yields also $\widehat{\mathbf{I}}_l$ in the encoder through D_l (center upper part). $\widehat{\mathbf{I}}_l$ feeds the remaining compressing part. Such a choice reduces the distortion as \mathbf{I}_l is not available to the decompressing part but it also increases the numerical complexity as the remaining compressing part depends on the choice of q_l . \mathbf{I}_r (center of the upper part) represents the original right view. With $\widehat{\mathbf{I}}_l$, it is used by the Disparity Estimator (DE) to yield a disparity map denoted \mathbf{d} using the well-known BM algorithm. \mathbf{d} is then used by the Image Predictor (IP) to transform $\widehat{\mathbf{I}}_l$ into the predicted view, denoted \mathbf{I}_p .

More specifically, $\widehat{\mathbf{I}}_l$ and \mathbf{I}_r are decomposed into K non-overlapping blocks of same size. The upper left corner of the k -block is indicated by coordinates (i_k, j_k) . The pixels contained in the k -block are referred to by $(i_k + \Delta i, j_k + j)$ where $(\Delta i, j)$ spans \mathcal{B} , a set listing all internal-block displacements (including $(0,0)$).

\mathbf{d} is an array of K disparity values denoted as d_1, \dots, d_K . It describes the K right horizontal-shifts by which, in the IP-block, each $\widehat{\mathbf{I}}_l$ -block is transformed into an \mathbf{I}_p -block:

$$\mathbf{I}_p \left(\begin{bmatrix} i_k + \Delta i, \\ j_k + j \end{bmatrix} \right) = \widehat{\mathbf{I}}_l \left(\begin{bmatrix} i_k + \Delta i, \\ j_k + j + d_k \end{bmatrix} \right), \quad (3.2)$$

where k ranges from 1 to K and $(\Delta i, j)$ spans \mathcal{B} . This IP-block is shown on the upper right part in Fig. 3.1. To simplify notations, we do not indicate here the \mathbf{d} -dependency of \mathbf{I}_p .

The BM algorithm, in the DE block, consists in selecting for each k -block, the disparity value d_k for which the k -block \mathbf{I}_p -values resemble most the k -block \mathbf{I}_r -values in the sense that the mean squared error is minimized as follows:

$$d_k(q_l) = \arg \min_{d \in \mathbf{S}} \sum_{(\Delta i, j) \in \mathcal{B}} \left(\hat{\mathbf{I}}_l \begin{bmatrix} i_k + \Delta i \\ j_k + j + d \end{bmatrix} - \mathbf{I}_r \begin{bmatrix} i_k + \Delta i \\ j_k + j \end{bmatrix} \right)^2, \quad (3.3)$$

where \mathbf{S} contains all allowed disparity values. As $\hat{\mathbf{I}}_l$ is q_l -dependent, the disparity value found, d_k is also q_l -dependent. C (center upper part) is a lossless encoding operation of the disparity map \mathbf{d} . The resulting bit stream is transmitted to the decoder (center downward arrow connecting the dashed line) which recovers the exact disparity map \mathbf{d} , through D , being the inverse operation of C as follows:

$$\mathbf{d} = D(C(\mathbf{d})). \quad (3.4)$$

The recovered disparity map is used with $\hat{\mathbf{I}}_l$ by the second IP-block to yield according to Eq. (3.2), \mathbf{I}_p , this time in the decoder. This second IP-block is at the bottom in Fig. 3.1. \mathbf{R} (upper right corner) represents the residual image, that is the difference between the original right view and its prediction:

$$\mathbf{R} = \mathbf{I}_r - \mathbf{I}_p. \quad (3.5)$$

C_{q_r} (upper right corner) is a lossy encoding operation where $q_r \in \mathbf{Q}_r$ is its quality factor and \mathbf{Q}_r is the set of all allowed values. C_{q_r} compresses \mathbf{R} into a bit stream transmitted to the decoder (right downward arrow connecting the dashed line). D_r , being the inverse operation of C_{q_r} , is used in the decoder to get an approximation of \mathbf{R} denoted $\hat{\mathbf{R}}$. By reversing Eq.(3.5), the decoder gets an approximation of \mathbf{I}_r denoted as $\hat{\mathbf{I}}_r$ and given by:

$$\hat{\mathbf{I}}_r = \mathbf{I}_p + D_r(C_{q_r}(\mathbf{R})). \quad (3.6)$$

In general, $\hat{\mathbf{I}}_r$ is closer to \mathbf{I}_r than \mathbf{I}_p and this improvement of \mathbf{I}_p is being referred to as *compensation*.

The bitrate, denoted b , is deduced from the bit streams $C_{q_l}(\mathbf{I}_l)$, $C(\mathbf{d})$ and $C_{q_r}(\mathbf{R})$:

$$b(\mathbf{I}_l, \mathbf{d}, \mathbf{I}_r, q_l, q_r) = \frac{|C_{q_l}(\mathbf{I}_l)| + |C(\mathbf{d})| + |C_{q_r}(\mathbf{R})|}{|\mathbf{I}_l| + |\mathbf{I}_r|}, \quad (3.7)$$

where $|\cdot|$ is the set cardinal number, here it helps counting, above, the number of bits, and below, the number of pixels.

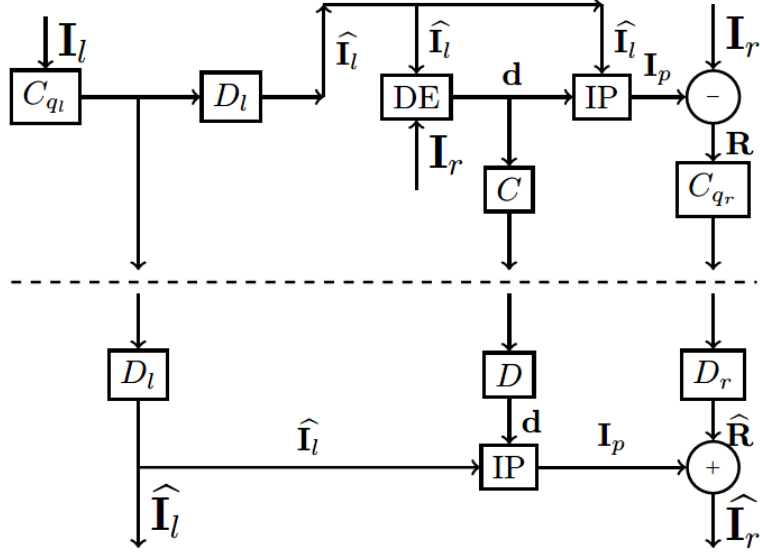


Figure 3.1: Disparity compensated coding scheme where the encoder (above) is separated from the decoder (below) by a dashed line.

3.1.2 Optimization problem statement

The aim of a coding/decoding scheme is a trade-off between getting the highest quality (e.g. visual rendering) while using the least amount of bits accounted for by Eq. (3.7). Here this trade-off is rephrased into finding the best quality (i.e. visual rendering) within a constrained bit-budget. As the end user observing the reconstructed stereoscopic image is generally a human being, our focus should be the extent to which the visual experience is being preserved. Designing objective quality metric modelling the quality of this visual experience is an existing research field as exemplified in [44]. As of now, no objective quality metric has proven to be completely reliable when applied to stereoscopic images. We are considering a quality metric, denoted as J to be used as a cost function in an optimization problem. It is for the sake of simplifying the description of the proposed algorithm is in section 3.2. As a consequence, at the coder level the choice of q_l, q_r, \mathbf{d} should within a constrained bit-budget minimize J . Generally, the

cost function of an image \mathbf{I}' as compared to an image \mathbf{I} is:

$$J(\mathbf{I}', \mathbf{I}) = \frac{1}{K} \sum_{k=1}^K J_k(\mathbf{I}', \mathbf{I}). \quad (3.8)$$

where J_k is the cost function of the k -block.

Numerical complexity being an important issue, the two following assumptions seem to be required to make this optimization problem tractable:

- A1) *Up to a non decreasing or non-increasing mapping, J is the sum of a metric computed independently on each blocks of $\widehat{\mathbf{I}}_r$ and \mathbf{I}_r and a metric computed on $\widehat{\mathbf{I}}_l$ and \mathbf{I}_l .*
- A2) *The bitrate control is addressed solely by selecting q_l and q_r .*

With these assumptions and denoting b_a the allowed bit-budget, the optimization problem addressing the optimal choice of q_l, q_r, \mathbf{d} at the coder level is:

$$\mathbf{d}(q_l, q_r) = \arg \min_{\mathbf{d} \in \mathbf{S}^K} J(\widehat{\mathbf{I}}_r(q_l, q_r, \mathbf{d}), \mathbf{I}_r) \quad (3.9)$$

$$\mathcal{J}(\widehat{\mathbf{I}}_l, \mathbf{I}_l, \widehat{\mathbf{I}}_r, \mathbf{I}_r) = \frac{1}{2} J(\widehat{\mathbf{I}}_l, \mathbf{I}_l) + \frac{1}{2} J(\widehat{\mathbf{I}}_r, \mathbf{I}_r) \quad (3.10)$$

$$(q_l, q_r) = \arg \min_{q_l \in \mathbf{Q}_l, q_r \in \mathbf{Q}_r, b \leq b_a} \mathcal{J}(\widehat{\mathbf{I}}_l(q_l), \mathbf{I}_l, \widehat{\mathbf{I}}_r(q_l, q_r, \mathbf{d}(q_l, q_r)), \mathbf{I}_r), \quad (3.11)$$

where b , defined in Eq. (3.7), depends on $\mathbf{I}_l, \mathbf{d}, \mathbf{I}_r, q_l, q_r$. \mathbf{S}^K is the set of all arrays of size K whose components are in \mathbf{S} and b_a is the expected bitrate.

Investigating the link between the BM algorithm and this optimization problem, Eq. (3.3) is recasted into:

$$d_k(q_l) = \arg \min_{s \in \mathbf{S}} J_k(\mathbf{I}_p, \mathbf{I}_r). \quad (3.12)$$

When considering the whole array of disparities, Eq. (3.12) becomes:

$$\mathbf{d}(q_l) = \arg \min_{\mathbf{d} \in \mathbf{S}^K} J(\mathbf{I}_p, \mathbf{I}_r). \quad (3.13)$$

Let us now suppose that the objective quality metric chosen is the PSNR (Peak

Signal to Noise Ratio) whose specific definition for stereoscopic images is:

$$\text{PSNR}(\mathbf{I}_l, \widehat{\mathbf{I}}_l, \mathbf{I}_r, \widehat{\mathbf{I}}_r) = 10 \log_{10} \left(\frac{255^2(|\mathbf{I}_l| + |\mathbf{I}_r|)}{\sum_{i,j} (\widehat{\mathbf{I}}_l(i,j) - \mathbf{I}_l(i,j))^2 + \sum_{i,j} (\widehat{\mathbf{I}}_r(i,j) - \mathbf{I}_r(i,j))^2} \right), \quad (3.14)$$

where pixel values are ranging from 0 to 255, i, j span all pixel positions and $|\mathbf{I}_l|, |\mathbf{I}_r|$ denote the number of pixels.

Hence, the BM algorithm can be regarded as a suboptimal solution of Eq. (3.10), where the effect of the choice of the disparity on the residual, and the residual impact on the distortion, are neglected. Note that from then on, this DCC algorithm is referred to as BM algorithm. Its description is summarized in Algorithm 1 even when the quality metric is not the PSNR (e.g SSIM) .

Algorithm 1 BM algorithm

Input: $\mathbf{I}_l, \mathbf{I}_r, q_l, q_r$

Output: $C_{q_l}(\mathbf{I}_l), C(\mathbf{d}), C_{q_r}(\mathbf{R}), b, \mathcal{J}$

Compute $C_{q_l}(\mathbf{I}_l), \widehat{\mathbf{I}}_l$ with Eq. (3.1) and $J(\widehat{\mathbf{I}}_l, \mathbf{I}_l)$ with Eq. (3.8)

for all $k \in \{1 \dots K\}$ **do**

for all $d \in \mathbf{S}$ **do**

 Compute the k -block of \mathbf{I}_p with Eq. (3.2) and $J_k(\mathbf{I}_p, \mathbf{I}_r)$

end for

 Select d_k with Eq. (3.12) minimizing $J_k(\mathbf{I}_p, \mathbf{I}_r)$

end for

Collect $\mathbf{d} = (d_1, \dots, d_K)$ and compute $C(\mathbf{d})$

Compute \mathbf{I}_p with Eq. (3.2), \mathbf{R} and $C_{q_r}(\mathbf{R})$ with Eq. (3.5)

Compute $\widehat{\mathbf{I}}_r$ with Eq. (3.6) and $J(\widehat{\mathbf{I}}_r, \mathbf{I}_r)$ with Eq. (3.8)

Compute \mathcal{J} with Eq. (3.10) using $J(\widehat{\mathbf{I}}_l, \mathbf{I}_l)$ and $J(\widehat{\mathbf{I}}_r, \mathbf{I}_r)$

Compute $b(\mathbf{I}_l, \mathbf{d}, \mathbf{I}_r, q_l, q_r)$ with Eq. (3.7) using $C_{q_l}(\mathbf{I}_l), C(\mathbf{d}), C_{q_r}(\mathbf{R})$

3.2 Proposed Disparity-Compensated Block Matching algorithm

The proposed Disparity-Compensated Block Matching algorithm, denoted as DCBM algorithm, is different from the BM algorithm in that Eq. (3.12) is no longer simplified into Eq. (3.13). DCBM algorithm is derived from a different suboptimal solution involving much greater numerical complexity.

The DCBM algorithm is computed in $K + 1$ steps. In the first step, the disparity map is computed using the BM algorithm. This initial disparity map has the K following

components:

$$d_k(0, q_l) = \arg \min_{d \in S} J_k(\mathbf{I}_p, \mathbf{I}_r), \quad (3.15)$$

where k ranges from 1 to K . Note that at this point $\mathbf{d}(0, q_l)$ does not depend on q_r .

The goal at step $t \in \{1, \dots, K\}$ is to select the k -block disparity, denoted, for now, as s . We assume that a disparity map $\mathbf{d}(t-1, q_l, q_r)$ has already been computed at step $t-1$. For each $s \in \mathbf{S}$, a predicted image $\mathbf{I}_p(t, q_l, q_r, s)$ is computed taking into account s on the t^{th} block and $d_k(t-1, q_l, q_r)$ for all other blocks:

$$\mathbf{I}_p(t, q_l, q_r, s) \begin{bmatrix} i_k + \Delta i \\ j_k + j \end{bmatrix} = \begin{cases} \widehat{\mathbf{I}}_l \begin{bmatrix} i_k + \Delta i \\ j_k + j + d_k(t-1, q_l, q_r) \end{bmatrix} & \text{if } k \neq t \\ \widehat{\mathbf{I}}_l \begin{bmatrix} i_k + \Delta i \\ j_k + j + s \end{bmatrix} & \text{if } k = t \end{cases} \quad (3.16)$$

with $(\Delta i, j)$ spanning \mathcal{B} and k ranging from 1 to K .

Compensation transforms $\mathbf{I}_p(t, q_l, q_r, s)$ into $\widehat{\mathbf{I}}_r(t, q_l, q_r, s)$ as follows:

$$\widehat{\mathbf{I}}_r(t, q_l, q_r, s) = \mathbf{I}_p(t, q_l, q_r, s) + D_r C_{q_r} (\mathbf{I}_r - \mathbf{I}_p(t, q_l, q_r, s)). \quad (3.17)$$

Finally $J(\widehat{\mathbf{I}}_r, \mathbf{I}_r)$ is computed and the best disparity is selected as follows:

$$d_k(t, q_l, q_r) = \begin{cases} d_k(t-1, q_l, q_r) & \text{if } k \neq t \\ \arg \min_{s \in \mathbf{S}} J(\widehat{\mathbf{I}}_r(t, q_l, q_r, s), \mathbf{I}_r) & \text{if } k = t \end{cases} \quad (3.18)$$

The DCBM algorithm is summarized in algorithm 2.

Note that the increased numerical complexity when using DCBM, stems from the necessity, to code and decode a new image, at each block and then each time a new disparity value is considered.

Algorithm 2 DCBM algorithm

Input: $\mathbf{I}_l, \mathbf{I}_r, q_l, q_r$

Output: $C_{q_l}(\mathbf{I}_l), C(\mathbf{d}), C_{q_r}(\mathbf{R}), b, \mathcal{J}$

Compute $C_{q_l}(\mathbf{I}_l), \hat{\mathbf{I}}_l$ with Eq. (3.1) and $J(\hat{\mathbf{I}}_l, \mathbf{I}_l)$ with Eq. (3.8)

Compute $\mathbf{d}(0, q_l)$ with Eq. (3.15) using \mathbf{I}_p defined by Eq. (3.2)

for all $t \in \{1 \dots K\}$ **do**

for all $s \in \mathbf{S}$ **do**

 Compute $\mathbf{I}_p(t, q_l, q_r, s)$ with Eq. (3.16) using $\mathbf{d}(t-1, q_l, q_r)$

 Compute $\hat{\mathbf{I}}_r(t, q_l, q_r, s)$ with Eq. (3.17)

 Compute $J(\hat{\mathbf{I}}_r(t, q_l, q_r, s), \mathbf{I}_r)$ with Eq. (3.8)

end for

 Select $\mathbf{d}(t, q_l, q_r)$ with Eq. (3.18) using all s -values of $J(\hat{\mathbf{I}}_r, \mathbf{I}_r)$

end for

Get $\mathbf{d} = \mathbf{d}(K, q_l, q_r)$ and compute $C(\mathbf{d})$

Compute \mathbf{I}_p with Eq. (3.2) using \mathbf{d}

Compute $\mathbf{R} = \mathbf{I}_r - \mathbf{I}_p$ and $C_{q_r}(\mathbf{R})$ with Eq. (3.5)

Compute $\hat{\mathbf{I}}_r$ with Eq. (3.6) and $J(\hat{\mathbf{I}}_r, \mathbf{I}_r)$ with Eq. (3.8)

Compute \mathcal{J} with Eq. (3.10) using $J(\hat{\mathbf{I}}_l, \mathbf{I}_l)$ and $J(\hat{\mathbf{I}}_r, \mathbf{I}_r)$

Compute $b(\mathbf{I}_l, \mathbf{d}, \mathbf{I}_r, q_l, q_r)$ with Eq. (3.7) using $C_{q_l}(\mathbf{I}_l), C(\mathbf{d}), C_{q_r}(\mathbf{R})$

3.3 Experiments and results

The performance of the proposed DCBM algorithm is compared to the BM and R [1] algorithms with a traditional compensation scheme on some stereoscopic images extracted from the Middlebury database [45] and the Deimos dataset [46].

In these experiments, the PSNR (defined in Eq. (3.14)) is used both as a performance measure and as the cost function used in the algorithms. To reduce the important amount of computations, the left view is not encoded and the PSNR is computed using only $\hat{\mathbf{I}}_r$ and \mathbf{I}_r with a simplified definition. The Structural Similarity Index (SSIM) [47] is also used to evaluate the performance. The bitrate in bit per pixel (bpp) takes into account the amounts of bits to encode \mathbf{d} and \mathbf{R} and Eq. (3.7) is modified into: $b(\mathbf{d}, \mathbf{I}_r) = \frac{|C(\mathbf{d})| + |C_{q_r}(\mathbf{R})|}{|\mathbf{I}_r|}$. Arithmetic coding [48] has been chosen to encode \mathbf{d} and JPEG (i.e. DCT and quantization) to encode the \mathbf{R} . To reduce the numerical complexity, the available range of JPEG hyperparameter values has been reduced in \mathbf{Q}_r to $\{10, 20, \dots, 90\}$. Of great importance is the choice of the block size, set to 8×8 as it matches the JPEG-block size. Pixel values are ranging from 0 to 255. The searching window \mathbf{S} is, unless otherwise specified, $\{-14, \dots, 15\}$.

Table 3.1 compares the performance of the DCBM algorithm to the BM algorithm using

Bjontegaard metric [49]. On 10 stereoscopic images, the table shows on the second column the average increased PSNR value at equal bitrate and on the third column the average bitrate decreasing ratio at equal distortion level. It appears that when using DCBM algorithm the increase in performance is substantial, above 1 dB on most tested images.

Table 3.1: Average increased performance of the DCBM algorithm compared to BM algorithm (with a traditional compensated scheme) using the Bjontegaard metric.

Image	Δ PSNR (dB)	bpp (%)
House	+5.69	-44.6
Art	+2.39	-37.9
Bowling1	+1.85	-20.8
Books	+1.65	-27.5
Rubik	+1.63	-60.1
Aloe	+1.19	-30.1
Wood	+1.14	-18.9
Teddy	+1.1	-21
Barn	+0.44	-13
Deimos_573-12	+0.1	-4.4

The rate distortion curves of the DCBM algorithm (magenta curve connecting circles), the BM algorithm (green curve connecting plus signs) and R algorithm (blue curve connecting circles) are provided on Fig. 3.2. These results have been carried out on the stereoscopic image "Bowling1", of size is 417×370 , where the original right view is shown on Fig. 3.4. The increase in the PSNR-value is important and fairly constant over a wide range of bitrates (0.4 bpp and 1 bpp). Note that, for this stereoscopic image, when the searching window is reduced to $\{-7, \dots, 8\}$, performance happen to be very similar, as shown on the right of Fig. 3.3.

Simulations have been done using Matlab in a Windows environment on a computer having the following characteristics: 3.7GHz, one processor, quad cores. Each point on Fig. 3.2 needs 0.8 seconds using the BM algorithm and about 4 hours using the DCBM algorithm.

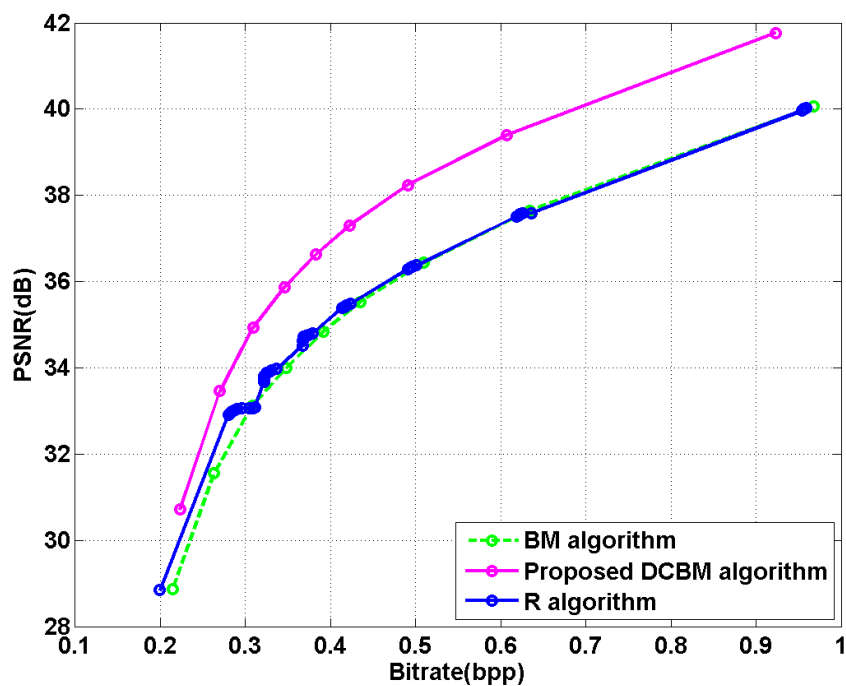


Figure 3.2: Comparison of DCBM, BM and R algorithms with $S = \{-14, \dots, 15\}$ on "Bowling1" stereoscopic image.

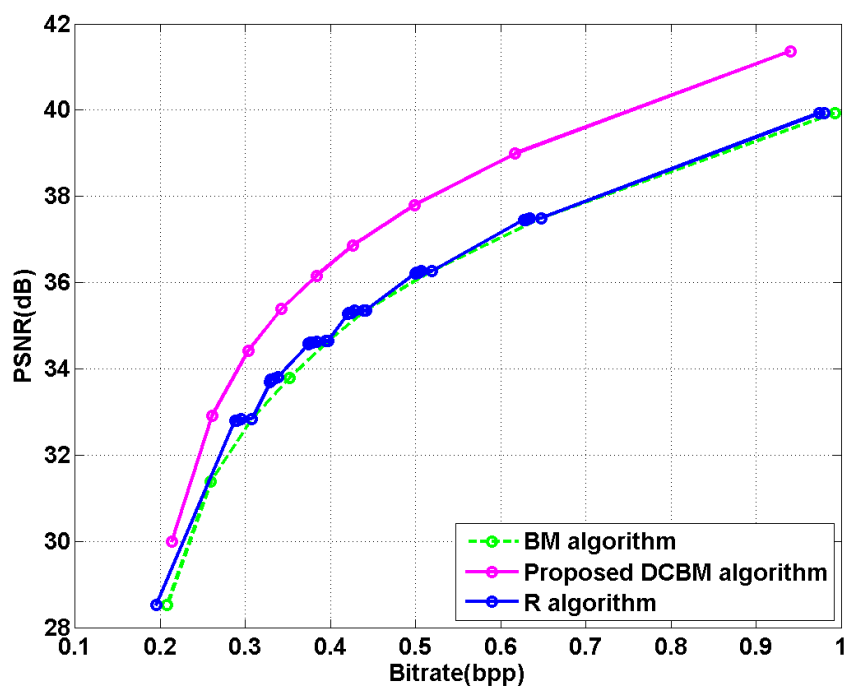


Figure 3.3: Comparison of DCBM, BM and R algorithms with $S = \{-7, \dots, 8\}$ on "Bowling1" stereoscopic image.

Fig. 3.5, Fig. 3.6 and Fig. 3.7 show respectively the reconstruction of the right view using the BM algorithm at a bitrate=0.38 bpp and SSIM=0.87, the R algorithm at a bitrate=0.39 bpp and SSIM=0.88 and DCBM algorithm at 0.38 bpp and SSIM=0.9. Differences between the three images can be observed when looking closely at the shadow, cast by the white left pin on the grey big ball: white block coding JPEG artifacts that can be seen on the BM-image and the R-image are no longer visible on the DCBM-image, even though JPEG is also used in the DCBM algorithm.



Figure 3.4: *Right view of the "Bowling1" stereoscopic image.*

Conclusion

This chapter presented a new stereoscopic image coding algorithm where the block-based disparity map is selected taking into account also the compensation effect (i.e. a refinement of the predicted image using the encoded residual image). As compared to the classical DC approach, a substantial increase in performance is observed on most tested stereoscopic images. However this comes at the cost of a huge increase in computational complexity, and this chapter proves the concept of the proposed strategy. An adequate modelling of JPEG distortion may prevent the increase in numerical complexity which is the objective of the next chapter.

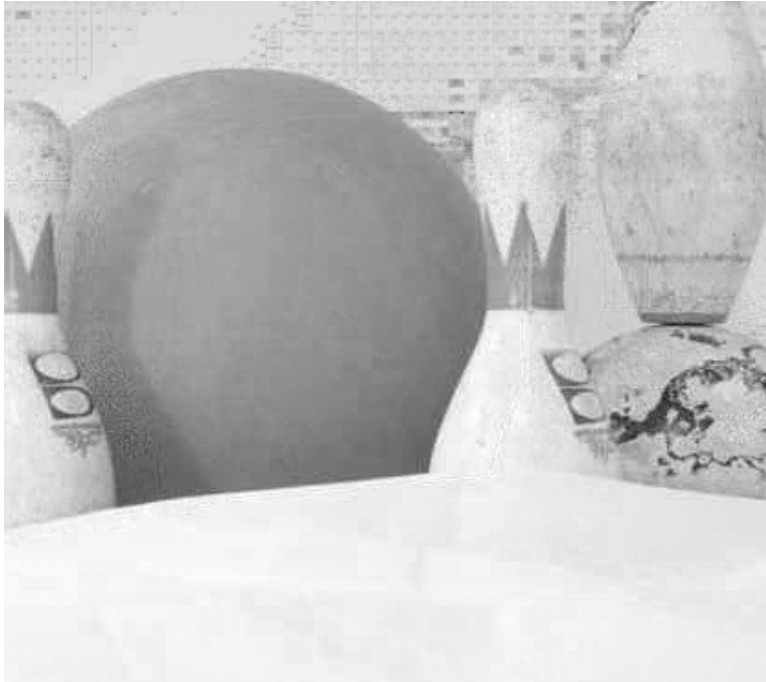


Figure 3.5: *Reconstruction using BM algorithm at $b = 0.39$ bpp.*

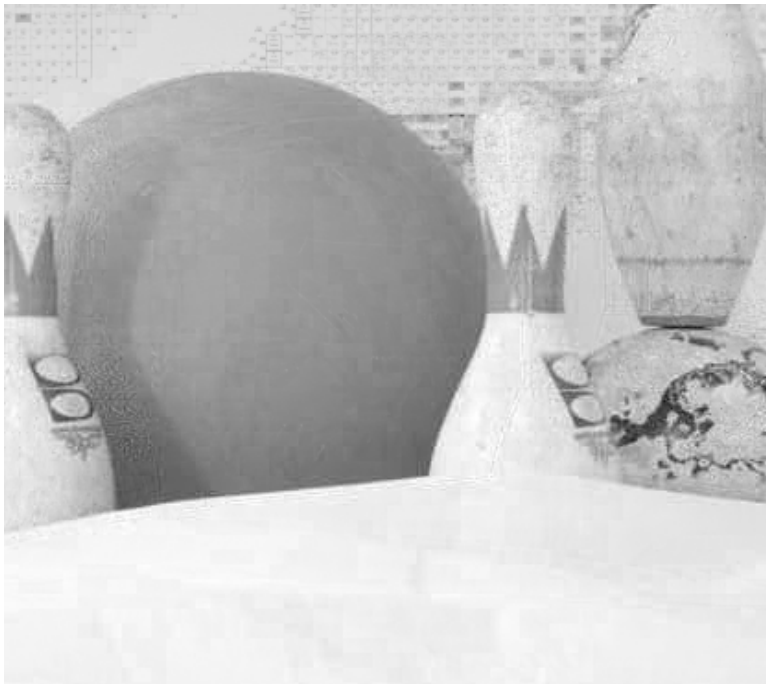


Figure 3.6: *Reconstruction using R algorithm at $b = 0.38$ bpp.*



Figure 3.7: *Reconstruction using DCBM algorithm at $b = 0.38$ bpp.*

Fast disparity map estimation algorithm improving stereoscopic image compression

Summary

In the previous chapter a proof of concept work has already shown that selecting disparities according to the compensated view, instead of the predicted view, yields increased rate-distortion performance. Due to its limitations in numerical complexity, this chapter proposes an algorithm deriving from the JPEG coder a disparity dependent analytic expression of the distortion induced by the compensated view.

The rest of this chapter is organized as follows. In Section 4.1, we recall the optimization problem. Then we describe the principles of the proposed algorithm in Section 4.2. In Section 4.3 evaluates the performance of the proposed Fast DCBM (FDCBM) algorithm given experimental results. Finally, some conclusions are drawn in Section 4.4. This contribution has resulted in one publication in an international journal [50].

4.1 Optimization problem

4.1.1 Basic concepts and notations

In this chapter we consider rectified stereoscopic images using the classical DCC scheme. The left image is chosen as the reference view and the right one as the target.

We keep the concepts and notations presented in Section 3.1.1 and illustrated in Fig. 3.1.

4.1.2 Formulation of the optimization problem

The optimization problem is described in Section 3.1.2. Furthermore, the mean squared error between $(\mathbf{I}_l, \hat{\mathbf{I}}_l)$ and $(\mathbf{I}_r, \hat{\mathbf{I}}_r)$, is used as the cost function to be minimized. More specifically, the mean squared error of the k -block of an image \mathbf{I}' as compared to that of an image \mathbf{I} is:

$$J_k(\mathbf{I}', \mathbf{I}) = \frac{1}{|\mathcal{B}|} \sum_{(\Delta i, \Delta j) \in \mathcal{B}} \left(\mathbf{I}' \begin{bmatrix} i_k + \Delta i \\ j_k + \Delta j \end{bmatrix} - \mathbf{I} \begin{bmatrix} i_k + \Delta i \\ j_k + \Delta j \end{bmatrix} \right)^2, \quad (4.1)$$

where $(\Delta i, \Delta j)$ spans the block \mathcal{B} , (i_k, j_k) design the coordinate of the upper left corner of the k -block and $||$ indicates the size of a set.

4.2 Proposed FDCBM algorithm

Due to the interesting performance of the DCBM algorithm (see [51]), this section proposes a Fast version of this algorithm called FDCBM algorithm. The novelty is that disparity selection is no longer based on the computation of $\hat{\mathbf{I}}_r$ with all its pixel values. The underlying idea of the developed algorithm is first discussed and then an explicit formula of the JPEG-codec distortion is derived. Blocks of size 8×8 pixels are considered knowing that an extension to a larger block size is possible.

4.2.1 FDCBM algorithm underlying idea

This section considers that the size of \mathcal{B} is 8×8 and more specifically that the disparity-related blocks are exactly the JPEG-related blocks.

Introduce first some new notations. Define $\hat{\mathbf{R}} = D_r C_{q_r}(\mathbf{R})$ the reconstructed residual at the decoder, and \mathbf{I}_k any matrix of size 8×8 :

$$\begin{cases} \mathbf{R}_k(\Delta i, j) &= \mathbf{R}(i_k + \Delta i, j_k + j) \\ \widehat{\mathbf{R}}_k(\Delta i, j) &= \widehat{\mathbf{R}}(i_k + \Delta i, j_k + j) \\ \|\mathbf{I}_k\|^2 &= \frac{1}{|\mathcal{B}|} \sum_{(\Delta i, j) \in \mathcal{B}} (\mathbf{I}_k(\Delta i, j))^2 \end{cases} \quad (4.2)$$

So as to be consistent with notations defined in Secion. 3.1.1, indexes of these 8×8 matrices start from 0: $\Delta i, \Delta j \in \{0, \dots, 7\}$. Note that because of the above block-related assumption, $\widehat{\mathbf{R}}_k$ can also be considered as the decoded-encoded 8×8 matrix \mathbf{R}_k :

$$\widehat{\mathbf{R}}_k = D_r C_{q_r}(\mathbf{R}_k). \quad (4.3)$$

Our main claim is that the relevant pixel values are those of \mathbf{R}_k , and that J_k measures the mean squared distortions yielded by the compression and decompression of \mathbf{R}_k :

$$\begin{aligned} J_k(\widehat{\mathbf{I}}_r, \mathbf{I}_r) &= J_k(\mathbf{I}_p + \widehat{\mathbf{R}}, \mathbf{I}_p + \mathbf{R}) = J_k(\widehat{\mathbf{R}}, \mathbf{R}) \\ &= \|D_r C_{q_r}(\mathbf{R}_k) - \mathbf{R}_k\|^2. \end{aligned} \quad (4.4)$$

The first equality is obtained with Eqs. (3.5) and (3.6). The second equality uses an additive invariance property derived from Eq. (4.1). The third equality is computed using Eqs. (4.1), (4.2) and (4.3).

4.2.2 JPEG encoding modelling

This section is interested in what JPEG encoding causes distortions, namely the quantization of the DCT components:

$$D_r C_{q_r}(\mathbf{R}_k) = \text{IDCT}[Q_{q_r}(\text{DCT}[\mathbf{R}_k])], \quad (4.5)$$

where Q_{q_r} is the 8×8 -JPEG-quantizier.

As DCT is an orthogonal transformation, it preserves the L2 norm:

$$\|D_r C_{q_r}(\mathbf{R}_k) - \mathbf{R}_k\|^2 = \|\text{DCT}[D_r C_{q_r}(\mathbf{R}_k)] - \text{DCT}[\mathbf{R}_k]\|^2. \quad (4.6)$$

Combining Eqs. (4.5) and (4.6), a minimized formula of the mean squared distortions

is obtained:

$$\|D_r C_{q_r}(\mathbf{R}_k) - \mathbf{R}_k\|^2 = \|Q_{q_r}(\mathbb{DCT}[\mathbf{R}_k]) - \mathbb{DCT}[\mathbf{R}_k]\|^2. \quad (4.7)$$

The explicit formula uses the following information extracted from the JPEG codec (see [52]). The DCT of an 8×8 matrix is:

$$\mathbb{DCT}[\mathbf{I}_k] = T^T \mathbf{I}_k T, \quad (4.8)$$

where T is an 8×8 orthogonal matrix defined as follows:

$$T_{\Delta i, j} = \frac{1}{\sqrt{8}} \cos\left(\pi \frac{(2j+1)\Delta i}{16}\right) \times \begin{cases} 1 & \text{if } \Delta i = 0 \\ \sqrt{2} & \text{if } 1 \leq \Delta i \leq 7 \end{cases} \quad (4.9)$$

The JPEG quantization table is given by Eq. (2.5).

The JPEG quantizer transforms an 8×8 matrix into an 8×8 matrix:

$$Q_{q_r}(\mathbf{I}) = \left[\text{Round}\left(\frac{\mathbf{I}(\Delta i, j)}{Q(\Delta i, j)\alpha(q_r)}\right) Q(\Delta i, j)\alpha(q_r) \right]_{\Delta i, j}, \quad (4.10)$$

using a nonlinear mapping transforms q_r into a scaling factor (see [53]):

$$\alpha(Q) = \begin{cases} \frac{50}{Q} & \text{if } Q \leq 50 \\ 2 - \frac{Q}{50} & \text{if } Q > 50 \end{cases} \quad (4.11)$$

Experimentations have shown that $J_k(\widehat{\mathbf{I}}_r, \mathbf{I}_r)$ is not exactly equal to the value of $\|Q_{q_r}(\mathbb{DCT}[\mathbf{R}_k]) - \mathbb{DCT}[\mathbf{R}_k]\|^2$, and the latter depends on q_l, q_r and on the k block disparity, s . So the following notation is used:

$$\tilde{J}_k(q_l, q_r, s) = \|Q_{q_r}(\mathbb{DCT}[\mathbf{R}_k]) - \mathbb{DCT}[\mathbf{R}_k]\|^2. \quad (4.12)$$

Finally the k block disparity is selected as:

$$d_k(q_l, q_r) = \arg \min_{s \in \mathbf{S}} \tilde{J}_k(q_l, q_r, s). \quad (4.13)$$

4.2.3 Derived FDCBM algorithm

Instead of computing large scale images as with DCBM algorithm, only 8×8 matrices are computed yielding to an approximation of $J_k(\widehat{\mathbf{I}}_r, \mathbf{I}_r)$ (i.e. $\tilde{J}_k(q_l, q_r, s)$) using Eq. (4.12). Moreover, instead of selecting the k -block disparity based on $J(\widehat{\mathbf{I}}_r, \mathbf{I}_r)$, it is based

on the minimization of $\tilde{J}_k(q_l, q_r, s)$. The numerical complexity of FDCBM algorithm is then definitely much lower than that of DCBM algorithm. It remains higher than the BM algorithm, not only because of the complexity of Eq. (4.12) but also because it takes into account q_l and q_r , whereas BM takes into account only q_l . The FDCBM algorithm is summarized in Algorithm 3.

Algorithm 3 FDCBM algorithm

Input: $\mathbf{I}_l, \mathbf{I}_r, q_l, q_r$

Output: $C_{q_l}(\mathbf{I}_l), C(\mathbf{d}), C_{q_r}(\mathbf{R}), b, \mathcal{J}$

 Compute $C_{q_l}(\mathbf{I}_l), \hat{\mathbf{I}}_l$ with Eq. (3.1) and $J(\hat{\mathbf{I}}_l, \mathbf{I}_l)$ with Eqs. (4.1) and (3.8)

for all $k \in \{1 \dots K\}$ **do**

for all $s \in \mathbf{S}$ **do**

 Compute \mathbf{R}_k using $\hat{\mathbf{I}}_l$ and \mathbf{I}_r with Eqs. (4.2), (3.2) and (3.5)

 Compute $\tilde{J}_k(q_l, q_r, s)$ with Eq. (4.12)

end for

 Select d_k with Eq. (4.13) using all s -values of $\tilde{J}_k(s)$

end for

 Collect $\mathbf{d} = (d_1, \dots, d_K)$ and compute $C(\mathbf{d})$

 Compute \mathbf{I}_p with Eq. (3.2) using \mathbf{d}

 Compute $\mathbf{R} = \mathbf{I}_r - \mathbf{I}_p$ and $C_{q_r}(\mathbf{R})$ with Eq. (3.5)

 Compute $\hat{\mathbf{I}}_r$ with Eq. (3.6) and $J(\hat{\mathbf{I}}_r, \mathbf{I}_r)$ with Eq. (3.8)

 Compute \mathcal{J} with Eq. (3.8) using $J(\hat{\mathbf{I}}_l, \mathbf{I}_l)$ and $J(\hat{\mathbf{I}}_r, \mathbf{I}_r)$

 Compute $b(\mathbf{I}_l, \mathbf{d}, \mathbf{I}_r, q_l, q_r)$ with Eq. (3.7) using $C_{q_l}(\mathbf{I}_l), C(\mathbf{d}), C_{q_r}(\mathbf{R})$

4.2.4 Extending the FDCBM algorithm to larger blocks

This section considers the case when the block decomposition yielding the disparity map is not the same than the JPEG-block decomposition. To distinguish them, the former is denoted \mathcal{B}_k , ($1 \leq k \leq K$, \mathcal{B} as the set of internal displacements), the latter is denoted $\mathcal{B}'_{k'}$, ($1 \leq k' \leq K'$, \mathcal{B}' as the set of internal displacements). In general, a block \mathcal{B}_k is likely to have common pixels with several blocks $\mathcal{B}'_{k'}$, and each of these blocks may have common pixels with other blocks $\mathcal{B}'_{k''}$. In such a situation, the optimal choice of a disparity d_k depends on the choice of disparities of neighboring blocks, and adapting the FDCBM algorithm seems difficult. Here we assume that each block \mathcal{B}_k can be divided exactly in a finite number of blocks $\mathcal{B}'_{k'}$, and show how FDCBM can easily be extended. For instance when an image is decomposed into 16×16 -blocks, each of them covers exactly four 8×8 -blocks. And when an image is decomposed into 32×32 -blocks, each of them covers exactly sixteen 8×8 -blocks.

Let \mathcal{K}_k be the set of indexes indicating the blocks $\mathcal{B}'_{k'}$ that are exactly covering \mathcal{B}_k :

$$\mathcal{B}_k = \bigcup_{k' \in \mathcal{K}_k} \mathcal{B}'_{k'}. \quad (4.14)$$

The size of \mathcal{K}_k can be computed:

$$\begin{cases} K|\mathcal{B}_k| &= K'|\mathcal{B}'_{k'}| \\ |\mathcal{B}_k| &= |\mathcal{K}_k||\mathcal{B}'_{k'}| \end{cases} \Rightarrow |\mathcal{K}_k| = \frac{K'}{K}, \quad (4.15)$$

where $||$ indicates the size of a set. The first equation is derived from the fact that each view of the considered stereoscopic image is divided into a set of non-overlapping blocks. The second equation is derived from equation (4.14).

The mean squared error defined in equation (4.1) has now two definitions depending on the considered block-decomposition:

$$\begin{aligned} J_k(\hat{\mathbf{I}}, \mathbf{I}) &= \frac{1}{|\mathcal{B}|} \sum_{(i,j) \in \mathcal{B}_k} (\hat{\mathbf{I}}(i,j) - \mathbf{I}(i,j))^2, \\ J'_{k'}(\hat{\mathbf{I}}, \mathbf{I}) &= \frac{1}{|\mathcal{B}'|} \sum_{(i,j) \in \mathcal{B}'_{k'}} (\hat{\mathbf{I}}(i,j) - \mathbf{I}(i,j))^2. \end{aligned} \quad (4.16)$$

Equation (4.14) yields a relationship between the two mean squared error functions:

$$J_k(\hat{\mathbf{I}}, \mathbf{I}) = \frac{1}{|\mathcal{K}_k|} \sum_{k' \in \mathcal{K}_k} J'_{k'}(\hat{\mathbf{I}}, \mathbf{I}). \quad (4.17)$$

Subsections 4.2.1 and 4.2.2 are applied to the JPEG-block decomposition, resulting in the following approximation:

$$J'_{k'}(\hat{\mathbf{I}}_r, \mathbf{I}_r) \approx \tilde{J}_{k'}(q_l, q_r, s), \quad (4.18)$$

where s is the k' -block disparity in the sense of the JPEG-block decomposition.

Equations (4.17) and (4.18) yield the k -block disparity:

$$d_k(q_l, q_r) = \arg \min_{s \in \mathbf{S}} \frac{1}{|\mathcal{K}_k|} \sum_{k' \in \mathcal{K}_k} \tilde{J}_{k'}(q_l, q_r, s). \quad (4.19)$$

This is the proposed extended FDCBM algorithm.

4.3 Performance of the proposed algorithm

This section starts with a discussion on the validity of Eq. (4.12) on which the proposed FDCBM algorithm is based. To do so, simulations are conducted on synthetic data to measure the ability of this equation to reduce distortions more than the BM algorithm.

For each $q_r \in \{1, \dots, 99\}$, 200 stereoscopic images of size 256×256 are randomly drawn from independent uniform distributions (left views are not encoded) using ω ranging from 1 to 200 and $\mathbf{S} = \{-14, \dots, 15\}$. On each image, a block is randomly selected and for this block, the BM, DCBM and FDCBM algorithms yield three disparities denoted as $d_{\text{BM}}(q_r, \omega)$, $d_{\text{DCBM}}(q_r, \omega)$, $d_{\text{FDCBM}}(q_r, \omega)$.

For each image and each algorithm, its mean squared distortion is computed and denoted as $J_k(q_r, d_{\text{BM}}(q_r, \omega), \omega)$, $J_k(q_r, d_{\text{DCBM}}(q_r, \omega), \omega)$ and $J_k(q_r, d_{\text{FDCBM}}(q_r, \omega), \omega)$. These simulations clearly confirm that:

$$\begin{cases} J_k(q_r, d_{\text{DCBM}}(q_r, \omega), \omega) \leq J_k(q_r, d_{\text{BM}}(q_r, \omega), \omega) \\ J_k(q_r, d_{\text{DCBM}}(q_r, \omega), \omega) \leq J_k(q_r, d_{\text{FDCBM}}(q_r, \omega), \omega) \end{cases}$$

Moreover, most often, simulations show that:

$$J_k(q_r, d_{\text{FDCBM}}(q_r, \omega), \omega) \leq J_k(q_r, d_{\text{BM}}(q_r, \omega), \omega).$$

To see how $J_k(q_r, d_{\text{FDCBM}}(q_r, \omega), \omega)$ is close to $J_k(q_r, d_{\text{DCBM}}(q_r, \omega), \omega)$ as compared to $J_k(q_r, d_{\text{BM}}(q_r, \omega), \omega)$, an average distortion reduction ratio is measured as follows:

$$\rho(q_r) = \frac{1}{200} \times \sum_{\omega=1}^{200} \frac{J_k(q_r, d_{\text{BM}}(q_r, \omega), \omega) - J_k(q_r, d_{\text{FDCBM}}(q_r, \omega), \omega)}{J_k(q_r, d_{\text{BM}}(q_r, \omega), \omega) - J_k(q_r, d_{\text{DCBM}}(q_r, \omega), \omega)}. \quad (4.20)$$

Fig. 4.1 illustrates the behaviour of the ratio $\rho(q_r)$ when q_r ranges from 1 to 100. When q_r is between 15 and 90, on average and compared to the distortions left when using BM algorithm, FDCBM algorithm is able to reduce at least 90% of the distortions that DCBM algorithm is able to reduce. The second part of this section concerns the simulation results performed on Middlebury dataset stereoscopic images [45], LIVE 3D dataset [54, 55] and Deimos database [46]. To simplify the experiment, the left view is not compressed. Assume that the pixel-values, on both views, are ranging from 0 to 255. The distortion of the predicted right view is measured using the Peak Signal to Noise

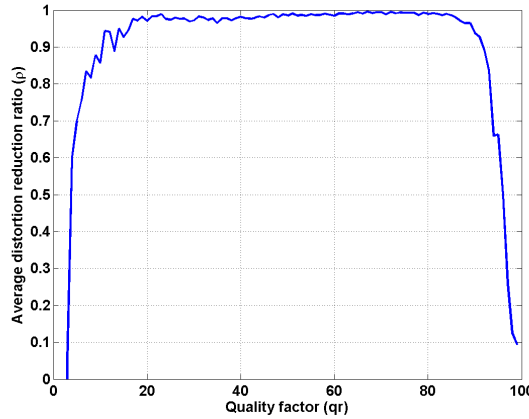


Figure 4.1: Average distortion reduction ratio of BM-FDCBM compared to BM-DCBM on synthetic data (function of q_r).

Ratio (*PSNR*) given by $PSNR = 10 \log_{10} \left(\frac{255^2}{J(\mathbf{I}_r, \mathbf{I}_r)} \right)$ and the Structural Similarity Index (SSIM) [47]. The rate, in bits per pixel (bpp), is measured only on the right view according to $b = \frac{|C(\mathbf{d})| + |C_{q_r}(\mathbf{R})|}{|\mathbf{I}_r|}$. The lossless coder, C is here an arithmetic coder (see [48]). To reduce the numerical complexity, the set of quality factor values is reduced to $\mathbf{Q}_r = \{5, 10, 15, \dots, 90\}$. The set of all available disparities is $\mathbf{S} = \{0, \dots, 120\}$.

The rate-distortion curves, provided in Fig. 4.2, confirm the results stated above using "Art" stereoscopic image of Middlebury-dataset (2005) and blocks of size 8×8 . Indeed the performance (in terms of rate-distortion) of the proposed FDCBM algorithm is similar to that of DBCM algorithm, which is however better than that of the classical BM algorithm and the Reference-based block matching algorithm called (R algorithm) proposed in [1]. Corresponding SSIM values when using FDCBM algorithm are given in Table 4.1 .

Fig. 4.3 presents the decompressed right image "Aloe" extracted from Middlebury dataset (2006) using BM algorithm on the left side, R algorithm in the middle and FDCBM on the right side. For each algorithm, blocks are of sizes 8×8 and $q_r \in \mathbf{Q}_r$ is set so that $b = 0.3$ bpp. When comparing both reconstructed views with the original view, it appears that the background cloth on right neighborhoods of each vertical leaf is wrongly drawn. The reason may be that these neighborhoods are occluded in the left view. The BM and R algorithms yield a dotted structure whereas the FDCBM algorithm yields a slightly blurred square texture. From a PSNR-viewpoint, the FDCBM-reconstructed view is closer to the original view (30.14dB) than the BM-reconstructed view (29.5dB)

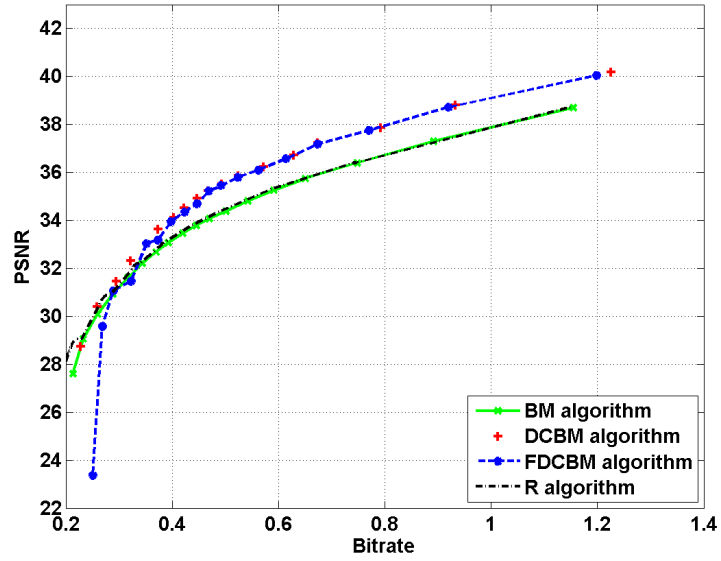


Figure 4.2: Performance comparison of BM, DCBM, FDCBM and R algorithms using "Art" stereoscopic image of Middlebury-dataset (2005).

Table 4.1: SSIM values using FDCBM algorithm using "Art" stereoscopic image.

Bitrate	SSIM
0.25	0.67
0.27	0.84
0.29	0.88
0.32	0.89
0.35	0.91
0.37	0.91
0.4	0.92
0.42	0.93
0.44	0.93
0.47	0.94
0.49	0.94
0.52	0.95
0.56	0.95
0.61	0.95
0.67	0.96
0.77	0.96
0.92	0.97
1.19	0.97

and the R-reconstructed image (29.6dB).

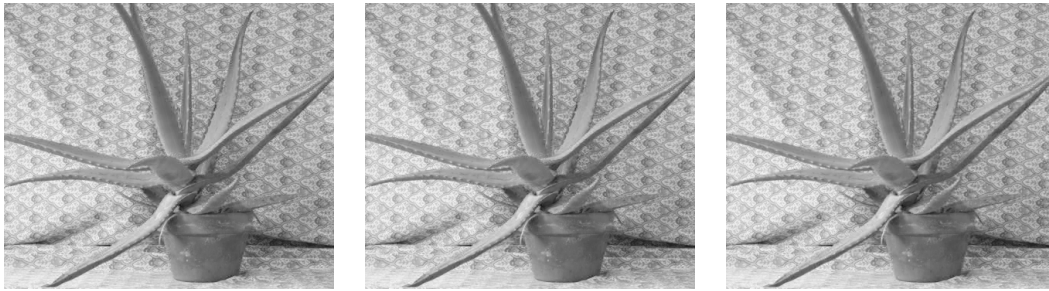


Figure 4.3: Reconstructed "Aloe" right view: BM algorithm (left side); R algorithm (middle) and FDCBM algorithm (right side).

Fig. 4.4 shows the histograms of, on the left side, the BM-disparity map on the mid side the R-disparity map, and on the right side, the FDCBM-disparity map for the same experiment. More specifically, selected disparity values are sorted into 10 bins, each bin is referred to by its average disparity value on the horizontal axis. The vertical axis indicates the number of blocks for which the disparity value falls into a given bin (the total number of blocks for that image is 2726). All histograms are right skewed, showing that for most blocks it did not proved useful to consider disparity values greater than 50. A closer look shows that, on the right hand side, the two first columns are slightly bigger and the two following columns are slightly smaller. This means that for this specific image, on average FDCBM algorithm tends to select smaller disparity values than BM and R algorithms.

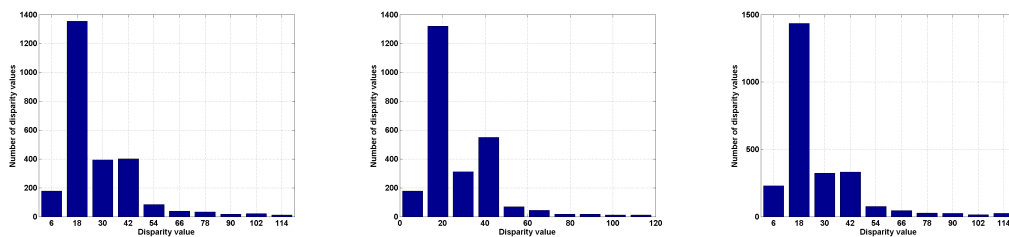


Figure 4.4: Histogram of the disparity map yielded at $b = 0.3\text{bpp}$ using "Aloe" stereoscopic image: BM algorithm (left side), R algorithm (on the middle) and FDCBM algorithm (right side).

Fig. 4.5 provides the reconstructions of the "Dwarves" right view from Middlebury-dataset 2005 using BM (in the left), R (on the middle) and FDCBM (on the right) algorithms. A zoom on the vase (bottom right), shows that FDCBM gives much closer reconstruction to the original view.



Figure 4.5: Reconstructed "Dwarves" right view: BM algorithm (left side); R algorithm (mid side) and FDCBM algorithm (right side).

Fig. 4.6 shows the original right view of "Deimos-550-1" stereo image from Deimos presented.

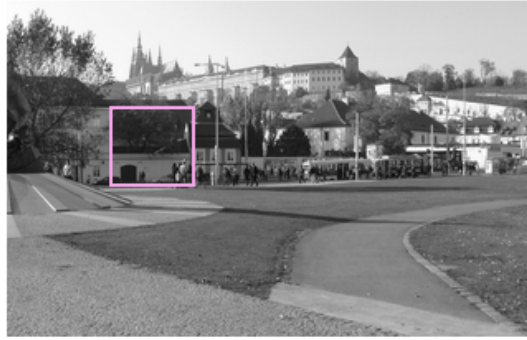


Figure 4.6: "Deimos-550-1" original right view.

When we look at Fig. 4.7 which illustrates three zoomed views of the reconstructed emphasized area, indicated by the the pink box in Fig. 4.6, using respectively BM(0.68bpp), FDCBM(0.65bpp) and R(0.68bpp) algorithms. It is visible that blocks artifact is reduced when using the proposed FDCBM compared to the two other algorithms.



Figure 4.7: Close-up views of the reconstructed "Deimos-550-1" right view: BM algorithm (left side); R algorithm (middle) and FDCBM algorithm (right side).

As for numerical complexity, FDCBM algorithm (consuming 17 seconds) is 3 388 times quicker than DCBM algorithm (consuming 4 hours), 6.8 times slower than BM

algorithm (consuming 2.5 seconds) and 1.5 times slower than R algorithm (consuming 12 seconds). This has been measured on the "Aloe" stereoscopic image with block of 8×8 size using Matlab in a Windows environment on a computer using one processor with four cores at a frequency of 3.7GHz.

The Bjøntegaard metric [49] is used here to quantify the increase in performance of FDCBM algorithm as compared to BM and R algorithms. Based on four rate-distortion points for each algorithm (roughly [0.3,0.4,0.5,0.6]bpp), it computes, an average PSNR increase, or, an average bitrate decrease. As for the "Art" stereoscopic image, FDCBM algorithm yields on average a PSNR increase of respectively 0.78dB and 0.52dB compared to the BM and R algorithms. To simplify its reading, the stereoscopic images have been sorted by their increase in PSNR-performance to compare FDCBM with BM algorithms.

Table 4.2 shows on columns 2 and 3 that, on average, for all stereoscopic images, FDCBM is better performing than BM, the difference ranges from 0.42dB up to 1.69dB. It seems difficult to understand why this difference is higher for some images and lower on other images. For instance "Cloth3" and "Cloth4" appear at both ends of the table and yet have similar appearance. The same comment applies to "Baby1" and "Baby3". And both "Midd1", "Midd2" and "Lampshade1", "Lampshade2" have similar appearance and yet each pair has quite different performance increases. It is interesting to note that the stereoscopic image having the least PSNR-performance increase (+0.17dB), namely "Plastic", is having a rather important bitrate decrease (-15.73%). Columns 4 and 5 of Table 4.2 summarizes the performance of our proposal compared to the R algorithm. One can observe that the FDCBM algorithm achieves better performance compared to the R algorithm. Table 4.3 provides other simulation results performed on LIVE 3D database [54, 55].

The increase in performance of all stereoscopic images of FDCBM as compared to BM is also shown in Fig. 4.8. More specifically, 10 bins have been considered ranging from +0.25dB up to +1.25dB in terms of increase in terms of PSNR performance of FDCBM as compared to BM. Each bar is associated to a specific bin, and its height indicates the number of stereoscopic images, having an increase in PSNR performance of roughly the amount indicated on the bin. The average increase in PSNR performance is 0.54dB.

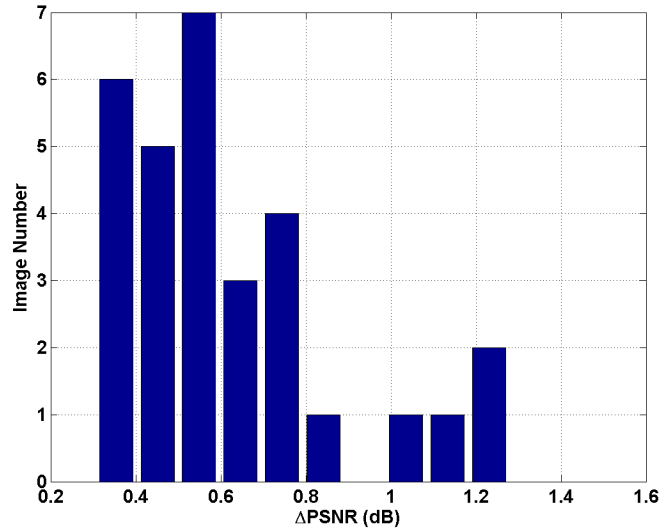


Figure 4.8: Histogram of the increase in average PSNR-performance of FDCBM as compared to BM, in terms of the number of stereoscopic images among the 30 extracted from the 2005 and 2006 Middlebury-database.

4.4 Conclusion

This chapter presents a new block-based disparity estimation technique called FDCBM algorithm. The purpose of this work is not to be competitive with stereoscopic image\video standards, but to first show the feasibility of the proposed approach as a proof of the concept. Where the classical technique selects each disparity so that the predicted image resembles most to the right view, the proposed technique computes for each disparity the compensated image, and the selected disparity is the one yielding the highest similarity between the compensated image and the right view. The computation is done with an analytic expression derived here from the JPEG-codec. To reduce the numerical complexity, these computations are fed using only the considered block pixel-values.

FDCBM algorithm is performing better than the classical Disparity Compensated Compression scheme using a Block Matching and Reference based block matching disparity estimation technique. To improve the performance of such proposal we can jointly consider the bitrate and distortion to estimate the disparity. This will be discussed in the next chapter.

Table 4.2: Performance comparison between FDCBM, BM and R algorithms using Bjøntegaard metric on Middlebury-datasets (2005-2006).

	FDCBM versus BM algorithms		FDCBM versus R algorithms	
Image	Δ PSNR (dB)	bpp (%)	Δ PSNR (dB)	bpp (%)
Plastic	+0.17	-15,73	+0.41	-10,3
Cloth3	+0.31	-7,44	+0.74	-12,78
Midd1	+0.31	-4,79	+0.1	-6,5
Cloth1	+0.34	-9,08	+0.2	-8.11
Laundry	+0.37	-5.66	+0.48	-7.61
Computer	+0.39	-6.3	+0.35	-6.09
Baby1	+0.39	-8.17	+0.42	-10.03
Baby2	+0.42	-9.35	+0.4	-7.44
Wood1	+0.43	-8.9	+0.68	-17.64
Rocks2	+0.44	-9.89	+0.49	-13.83
Books	+0.46	-7.93	+0.37	-5.61
Aloe	+0.53	-11.7	+0.6	-14.2
Lampshade1	+0.54	-5.09	+0.66	-13.01
Rocks1	+0.56	-12.43	+0.5	-13.53
Bowling2	+0.57	-9.86	+0.77	-13.94
Midd2	+0.58	-10.51	+0.11	-3.82
Drumsticks	+0.58	-8.85	+0.41	-6.67
Dolls	+0.59	-10.65	+0.51	-10.51
Moebius	+0.65	-11.44	+1.24	-22.59
Cloth2	+0.66	-12.68	+0.73	-15.47
Baby3	+0.67	-12.74	+0.55	-10.55
Wood2	+0.72	-9.34	+0.65	-8.42
Monopoly	+0.75	-13.56	+0.44	-6.53
Cloth4	+0.76	-17.12	+0.8	-14.63
Art	+0.78	-11.36	+0.52	-8.81
Bowling1	+0.89	-14.47	+0.11	-20.98
Dwarves	+1.06	-17.48	+0.64	-12.64
Flowerpots	+1.12	-14.52	+1.12	-16.52
Lampshade2	+1.23	-22.76	+0.91	-18.23
Mean	+0.62	-11.38	+0.54	-11.62

Table 4.3: Performance comparison between FDCBM, BM and R algorithms using the Bjøntegaard metric on 3D LIVE-database.

	FDCBM versus BM algorithms		FDCBM versus R algorithms	
Image	Δ PSNR (dB)	bpp (%)	Δ PSNR (dB)	bpp (%)
im8	+0.21	-5.85	+0.3	-1,76
im29	+0.22	-7.06	+0.15	-5.13
im13	+0.25	-7.43	+0.13	-4.03
im18	+0.34	-7.52	+0.15	-3.64
im3	+0.36	-11.14	+0.23	-7.68
im17	+0.51	-17.75	+0.4	-13.96
im10	+0.55	-14.22	+1.46	-8.5
im26	+0.61	-17.23	+0.51	-19.27
im5	+0.62	-18.26	+0.39	-11.34
Mean	+0.42	-12.74	+0.23	-8.37

Disparity estimation using a joint bitrate-distortion metric in the compensation coding scheme

Summary

CHALLENGING the exploitation of stereoscopic image redundancy to increase compression bitrate-distortion performance. In this chapter, an algorithm combining two existing compression algorithms is presented. These latter are different, one is performing better at low- and mid-range bitrates, it is the R-algorithm [1], the other at mid- and high-range bitrates, it is the FDCBM presented in the previous chapter. One iteratively modifies the disparity map to improve the bitrate-distortion trade-off using a Lagrangian multiplier. The other selects each disparity on a block basis, according to a simplified model of how JPEG deals with the compensation refinement. The remainder of this chapter is organized as follows. Section 5.1 recalls some notations and concepts related to the BM and FDCBM algorithms. Section 5.2 describes the R algorithm. Section 5.3 provides a description of the developed algorithm. In section 5.4, simulation results evaluating the proposed algorithm's performance is given. Finally, section 5.5 concludes this chapter. The main contribution, provided in this chapter, have been submitted to the international conference [56].

5.1 Notations and formulations

Let us recall some notations and formulations related to the BM and FDCBM [50] algorithms.

When a disparity map, of K blocks, $\mathbf{d} = (d_0, \dots, d_{K-1})$ is estimated using the BM algorithm, each component d_k ($k \in \{0, \dots, K\}$) is selected so that the k^{th} -block in the left view, is horizontally shifted by d_k , and best matches the k^{th} -block in the right view, in terms of mean square error D_k :

$$D_k(q_l, s) = \sum_{(\Delta i, \Delta j) \in \mathcal{B}} \left(\widehat{\mathbf{I}}_l \begin{bmatrix} i_k + \Delta i \\ j_k + \Delta j + s \end{bmatrix} - \mathbf{I}_r \begin{bmatrix} i_k + \Delta i \\ j_k + \Delta j \end{bmatrix} \right)^2, \quad (5.1)$$

where s is a disparity value taken from the search window S , $(\Delta i, \Delta j)$ spans the block \mathcal{B} , (i_k, j_k) design the coordinate of the upper left corner of the k^{th} -block, s is a disparity value, \mathbf{I}_r is the right view and $\widehat{\mathbf{I}}_l$ is the decoded left image using the q_l parameter.

The disparity d_k , estimated using the BM algorithm, is then selected as:

$$d_k^{(\text{BM})} = \underset{s \in \mathbf{S}}{\operatorname{argmin}} D_k(q_l, s). \quad (5.2)$$

As described in Section. 4.2, FDCBM algorithm overlooks the cost of storing the disparity map and selects the blockwise disparity taking into account the compensation refinement assuming that JPEG is used for compensation. The modeled compensation aware distortion is:

$$\widetilde{D}_k(q_l, q_r, s) = \|Q_{q_r}(\mathbb{DCT}[\mathbf{R}_k]) - \mathbb{DCT}[\mathbf{R}_k]\|^2, \quad (5.3)$$

where $\|\cdot\|^2$ is the $L2$ norm, \mathbf{R}_k designs the k - block of the residual error R , Q_{q_r} is the 8×8 JPEG quantizer, q_l and q_r are hyperparameters used respectively for coding the left view and the residual error and s is a disparity value taken for the k^{th} block.

The k -block disparity using FDCBM algorithm is given by:

$$d_k^{(\text{FDCBM})} = \underset{s \in \mathbf{S}}{\operatorname{argmin}} \widetilde{D}_k(q_l, q_r, s). \quad (5.4)$$

Note that the use of D in these formulas instead of J in previous chapters is to design distortion.

On the other hand the distortion of a stereoscopic image is given by:

$$\text{MSE} = \frac{1}{2} \|\mathbf{I}_r - \widehat{\mathbf{I}}_r\|^2 + \frac{1}{2} \|\mathbf{I}_l - \widehat{\mathbf{I}}_l\|^2. \quad (5.5)$$

with $\|\cdot\|^2$ is the $L2$ norm, I_r and \widehat{I}_r are respectively the original right view and its decoded version and I_l and \widehat{I}_l represent respectively the original left image and its decoded version.

5.2 Description of the R-algorithm

In essence, the R-algorithm, defined in [1], is a suboptimal solution to a disparity minimization problem overlooking the compensation refinement:

$$J(q_t, \mathbf{d}) = \frac{1}{K} \sum_{k=0}^{K-1} D_k(q_t, d_k) + \lambda H(\mathbf{d}), \quad (5.6)$$

where λ is a lagrangian multiplier. $H(\mathbf{d})$, modeling the bitrate, is the entropy derived from the empirical probability of the disparity map \mathbf{d} :

$$\left\{ \begin{array}{l} (\mathbf{d}, s) = \sum_{k=0}^{K-1} (d_k = s) \\ H(\mathbf{d}) = - \sum_{s \in \mathbf{S}} \frac{V(\mathbf{d}, s)}{K} \log_2 \left(\frac{V(\mathbf{d}, s)}{K} \right), \end{array} \right. \quad (5.7a)$$

$$\left\{ \begin{array}{l} H(\mathbf{d}) = - \sum_{s \in \mathbf{S}} \frac{V(\mathbf{d}, s)}{K} \log_2 \left(\frac{V(\mathbf{d}, s)}{K} \right), \end{array} \right. \quad (5.7b)$$

where $V(\mathbf{d}, s)$ designs the occurrence of s in \mathbf{d} and K is the total number of blocks.

The heuristic is a repeated raster scanning of each block, where the disparity is selected so as to minimize J . The block, whose disparity is being modified, is:

$$\Phi_t = t - K \left\lfloor \frac{t}{K} \right\rfloor \in \{0, \dots, K-1\}, \quad (5.8)$$

where t is a counter.

Let us define a block based entropy modeling the s -dependent impact on $H(\mathbf{d})$ of replacing d_k by s , and denote it, $\Delta H_k(\mathbf{d}, s)$:

$$\left\{ \begin{array}{l} V(\mathbf{d}, s) \log_2 \left(\frac{V(\mathbf{d}, s)}{K} \right) \\ - (V(\mathbf{d}, s) + 1) \log_2 \left(\frac{V(\mathbf{d}, s) + 1}{K} \right) \end{array} \right. \quad \text{if } s \neq d_k \quad (5.9a)$$

$$\left\{ \begin{array}{l} (V(\mathbf{d}, d_k) - 1) \log_2 \left(\frac{V(\mathbf{d}, d_k) - 1}{K} \right) \\ - V(\mathbf{d}, d_k) \log_2 \left(\frac{V(\mathbf{d}, d_k)}{K} \right) \end{array} \right. \quad \text{if } s = d_k \quad (5.9b)$$

As mentioned in [1], considering two disparity maps \mathbf{d} and \mathbf{d}' whose only difference is that $d'_k = s$ and $d_k = d_k$, we have:

$$\begin{aligned} H(\mathbf{d}') &= H(\mathbf{d}) + \Delta H_k(\mathbf{d}, s) \\ &\quad - (V(\mathbf{d}, d_k) - 1) \log_2 \left(\frac{V(\mathbf{d}, d_k) - 1}{K} \right) \\ &\quad + V(\mathbf{d}, d_k) \log_2 \left(\frac{V(\mathbf{d}, d_k)}{K} \right). \end{aligned} \quad (5.10)$$

Note that equations (5.1) and (5.10) show that minimizing J with respect to d_k is equivalent to minimizing $D_k(q_l, s) + \lambda \Delta H_k(\mathbf{d}, s)$.

The recast equations of the R-algorithm are:

$$\begin{aligned} \mathbf{d}^0 &= \mathbf{d}^{(\text{BM})}, \\ d_k^{t+1} &= \begin{cases} \underset{s \in \mathbf{S}}{\operatorname{argmin}} D_{\Phi_t}(q_l, s) + \lambda \Delta H_{\Phi_t}(\mathbf{d}^t, s) & \text{if } k = \Phi_t \\ d_k^t & \text{if } k \neq \Phi_t, \end{cases} \end{aligned} \quad (5.11)$$

where t is increased as long as $t \leq K$ and $\mathbf{d}^t \neq \mathbf{d}^{t-1}$. The yielded disparity map is denoted as $\mathbf{d}^{(R)}$, which depends on q_l and λ .

5.3 Proposed COMB-algorithm

Thanks to the new notations, combining the R and FDCBM algorithms is now straightforward, and our proposal is:

$$\begin{aligned} \mathbf{d}^0 &= \mathbf{d}^{(\text{FDCBM})}, \\ d_k^{t+1} &= \begin{cases} \underset{s \in \mathbf{S}}{\operatorname{argmin}} \tilde{D}_{\Phi_t}(q_l, q_r, s) + \lambda \Delta H_{\Phi_t}(\mathbf{d}^t, s) & \text{if } k = \Phi_t \\ d_k^t & \text{if } k \neq \Phi_t \end{cases} \end{aligned} \quad (5.12)$$

where t is increased as long as $t \leq K$ and $\mathbf{d}^t \neq \mathbf{d}^{t-1}$. The resulted disparity map is denoted as $\mathbf{d}^{(\text{COMB})}$.

The only differences between the COMB-algorithm and R-algorithm is the choice of the initial disparity, the 8×8 block size and the use of \tilde{D}_k instead of D_k . Note that since \mathbf{d}^t depends on q_l, q_r and λ , $\Delta H_{\Phi_t}(\mathbf{d}^t, s)$ depends also on these parameters.

The COMB-algorithm is described below (see Algorithm 4).

Algorithm 4 : *COMB – algorithm*

Input: $\mathbf{I}_l, \mathbf{I}_r$ **Output:** $b, \text{MSE}, \mathbf{d}$ for all q_l, q_r, λ

```
for all  $q_l$  do
  for all  $q_r$  do
    for all  $k \in \{0, \dots, K - 1\}$  do
      Compute  $d_k$  with (5.4)
    end for
    Collect  $d_k$  into  $\mathbf{d}^{(\text{FDCBM})}$ 
    for all  $\lambda$  do
      Continue = True,  $t = 0$ ,  $\mathbf{d}^{t-1} = \mathbf{d}^{(\text{FDCBM})}$ 
      while Continue is True do
        Continue =
        for all  $s \in \mathbf{S}$  do
          Compute  $\mathbf{d}^t$  with (5.12)
          if  $t \geq K$  and  $\mathbf{d}^t$  different from  $\mathbf{d}^{t-K}$  then
            Continue = True
          end if
        end for
      end while
      Store  $\mathbf{d}(q_l, q_r, \lambda) = \mathbf{d}^t$ 
      Compute  $\text{MSE}(q_l, q_r, \lambda)$  and  $b(q_l, q_r, \lambda)$ .
    end for
  end for
end for
Collect all  $\mathbf{d}, \text{MSE}, b$ .
```

5.4 Experiments and results

The BM, R, FDCBM and COMB algorithms have been tested on a set of stereoscopic images. This section provides the results using "*Monopoly*" stereoscopic image extracted from the 2006-Middlebury dataset [45] and "*Ballon*" stereoscopic image extracted from the MCL-3D dataset [57].

For the sake of simplicity, the left view is not coded. The bitrate is deduced as follows: $b = \frac{|C(\mathbf{d})| + |C_{qr}(\mathbf{R})|}{|\mathbf{I}_r|}$. The lossless compression of the disparity map is implemented using an entropy coder and JPEG is used to code the residual. The distortion is accounted for by $MSE = \|\mathbf{I}_r - \hat{\mathbf{I}}_r\|^2$. The PSNR and SSIM compare $\hat{\mathbf{I}}_r$ with \mathbf{I}_r .

Fig. 5.1 presents the rate-distortion curves obtained when compressing "*Monopoly*" using the four algorithms. The BM-curve in green is below with the R-curve in magenta slightly above it. Clearly above at high bitrate and below at low bitrate is the FDCBM-curve in blue. Above all these curves is the COMB-curve. What is striking, is that, even though by themselves, the R- and FDCBM algorithms are not doing that well, the COMB algorithm is significantly above at low- and mid-range bitrates.

Much the same can be said of Fig. 5.2 which is the rate-distortion curve obtained when compressing "*Ballon*". The increase in performance may seem quite surprising as the R-curve and the BM-curve are hardly distinguishable. The FDCBM algorithm has very low performance at low bitrate. And yet the COMB algorithm has still significant increase in compressing performance compare to the BM algorithm at low- and mid-range bitrates.

Fig. 5.4 provides the histograms of the disparity maps yielded by the four different algorithms when compressing "*Monopoly*" at $b = 0.33$. The cost of storing the disparity map is modeled with the entropy which is computed using such disparity histograms. Basically the more a disparity map has most of its values included in a small set of values, the lower is the entropy. With no surprise, histograms obtained with the R and COMB algorithms have much lower entropy as compared to those obtained with the BM and FDCBM algorithms.

The original right view of "*Monopoly*" stereoscopic image is depicted in Fig. 5.3.

Fig. 5.5 shows the right view reconstruction at $b = 0.33$ bpp when compression is achieved with the BM algorithm. On a computer screen with some zooming, white thin

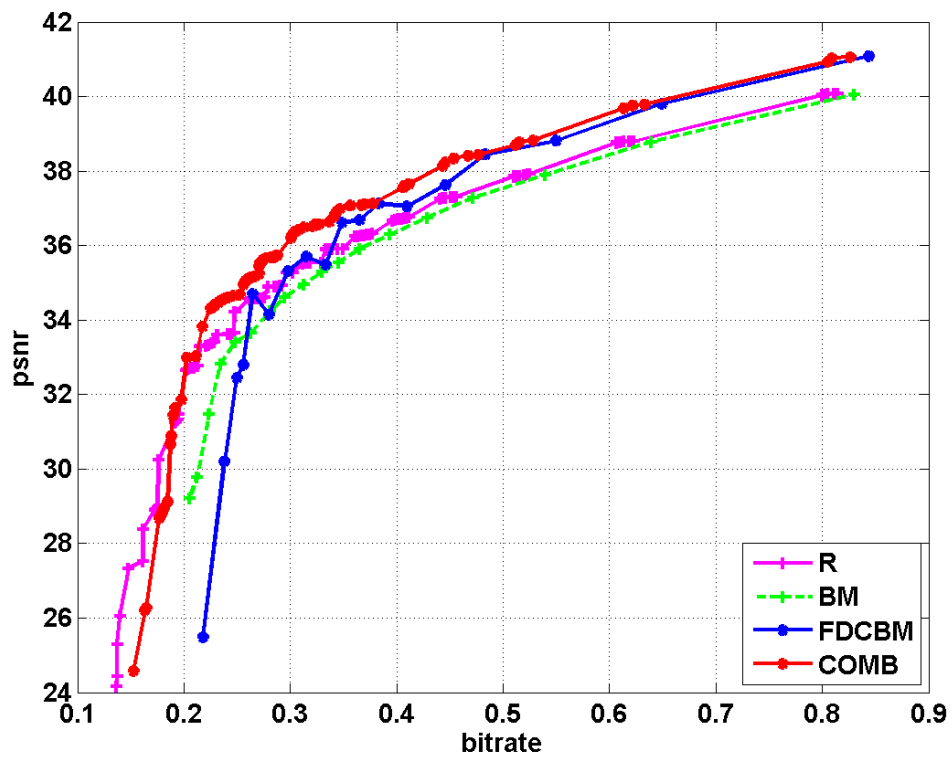


Figure 5.1: Performance comparison of BM, R, FDCBM and COMB algorithms on the "Monopoly" stereoscopic image .

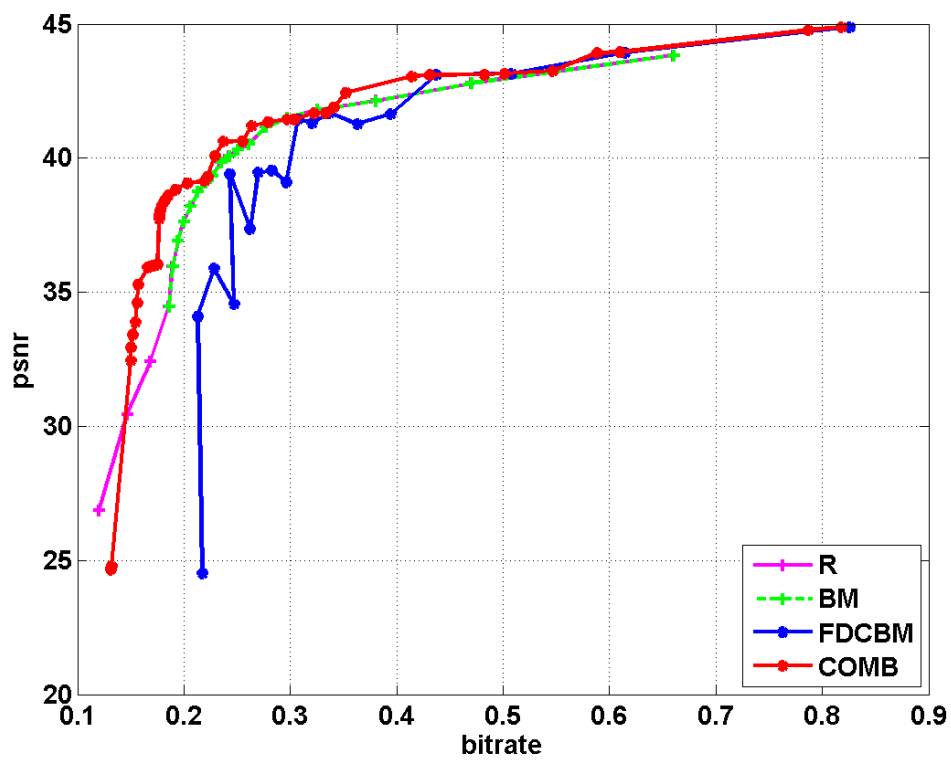


Figure 5.2: Performance comparison of BM, R, FDCBM and COMB algorithms using "Ballon" stereoscopic image.



Figure 5.3: "Monopoly" original right view.

vertical lines can be seen both at the right of the vertical thin piece of wood and at the right of the "Monopoly" board. These vertical lines are slightly less visible when compression is achieved with the R algorithm. However when compression is achieved with the FDCBM algorithm these white lines are hardly visible, they are replaced by gray squares of equal size and randomly padding the left, the right and the interior of the "Monopoly" board, such squares may be a little more visible than the vertical white lines. When compression is achieved with the COMB algorithm, the white lines remain hardly visible but the gray squares have also vanished assessing here an improvement.

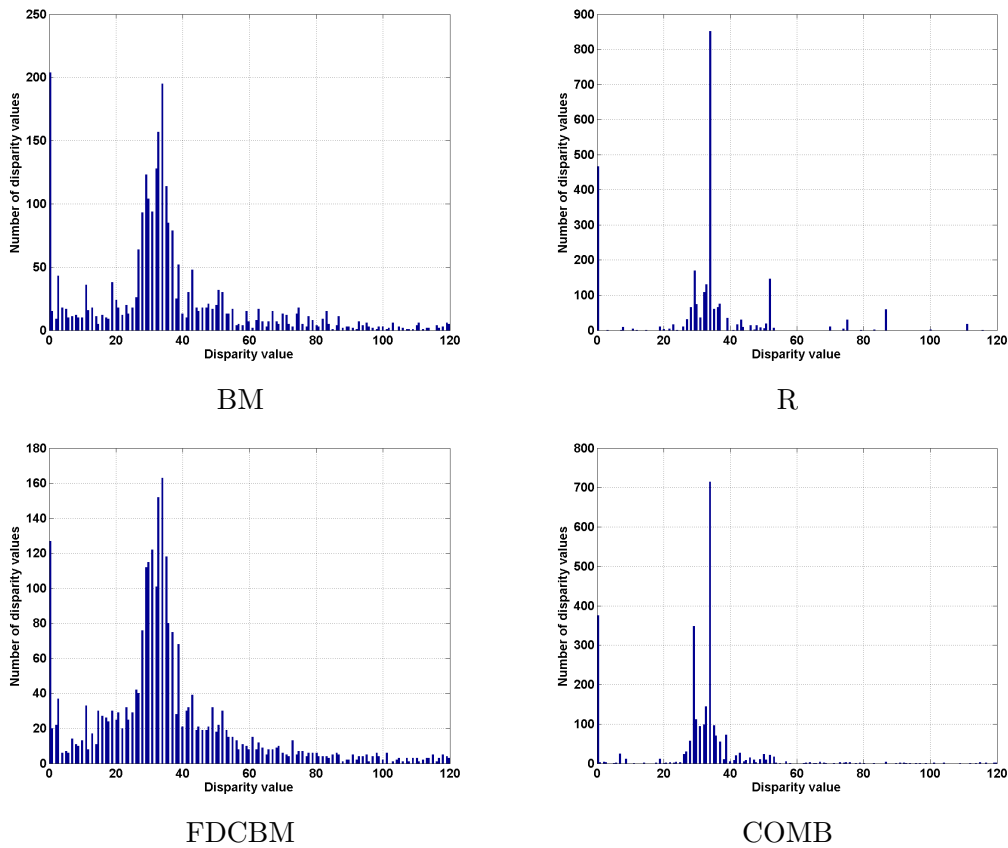


Figure 5.4: Histogram of the disparity map yielded at $b = 0.33$ bpp using Monopoly.

5.5 Conclusion

This chapter presented a new stereo-matching algorithm jointly optimizing the bitrate and the distortion. The proposed algorithm, called COMB-algorithm, combines the developed FDCBM algorithm and the Reference based block matching disparity estimation

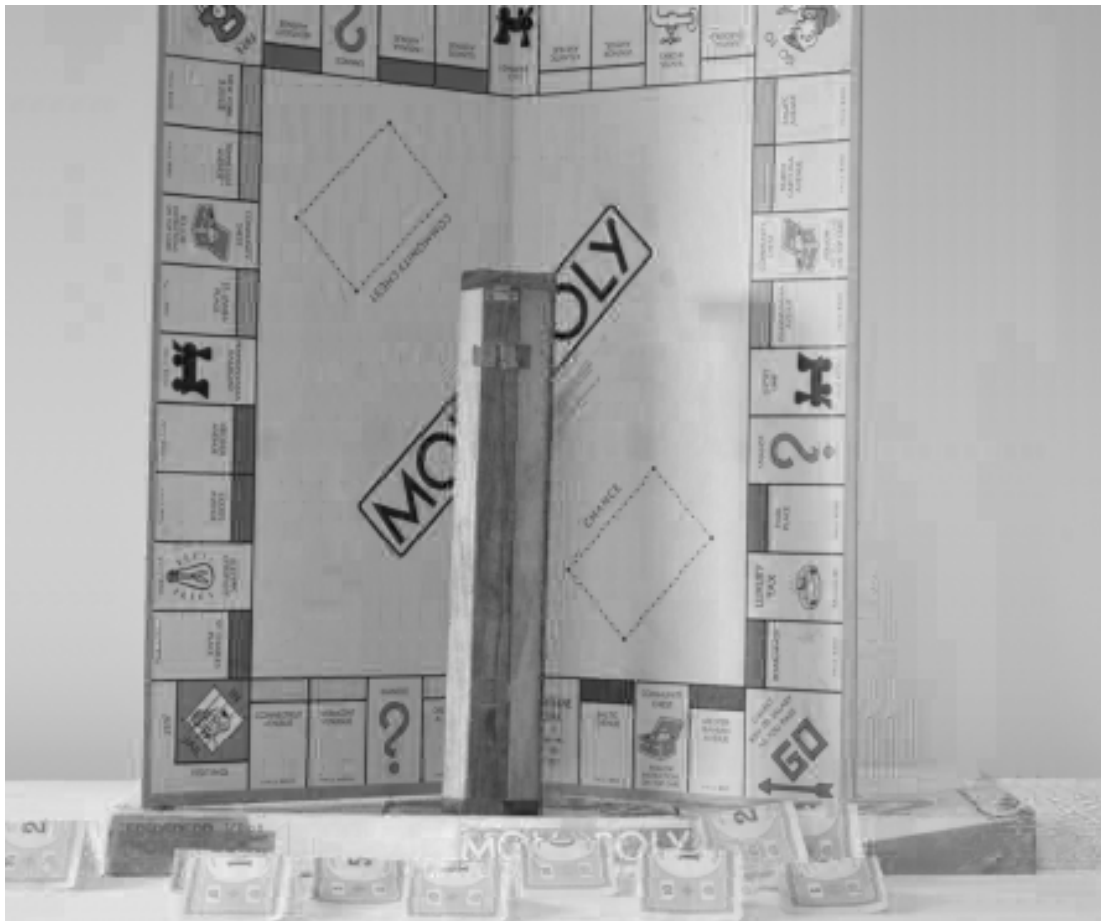


Figure 5.5: Reconstructed right view with the BM algorithm, ($PSNR = 35.28dB$, $SSIM = 0.95$, $b = 0.33bpp$).



Figure 5.6: *Reconstructed right view with the R algorithm (PSNR = 35.89dB, SSIM = 0.95, b = 0.33bpp).*

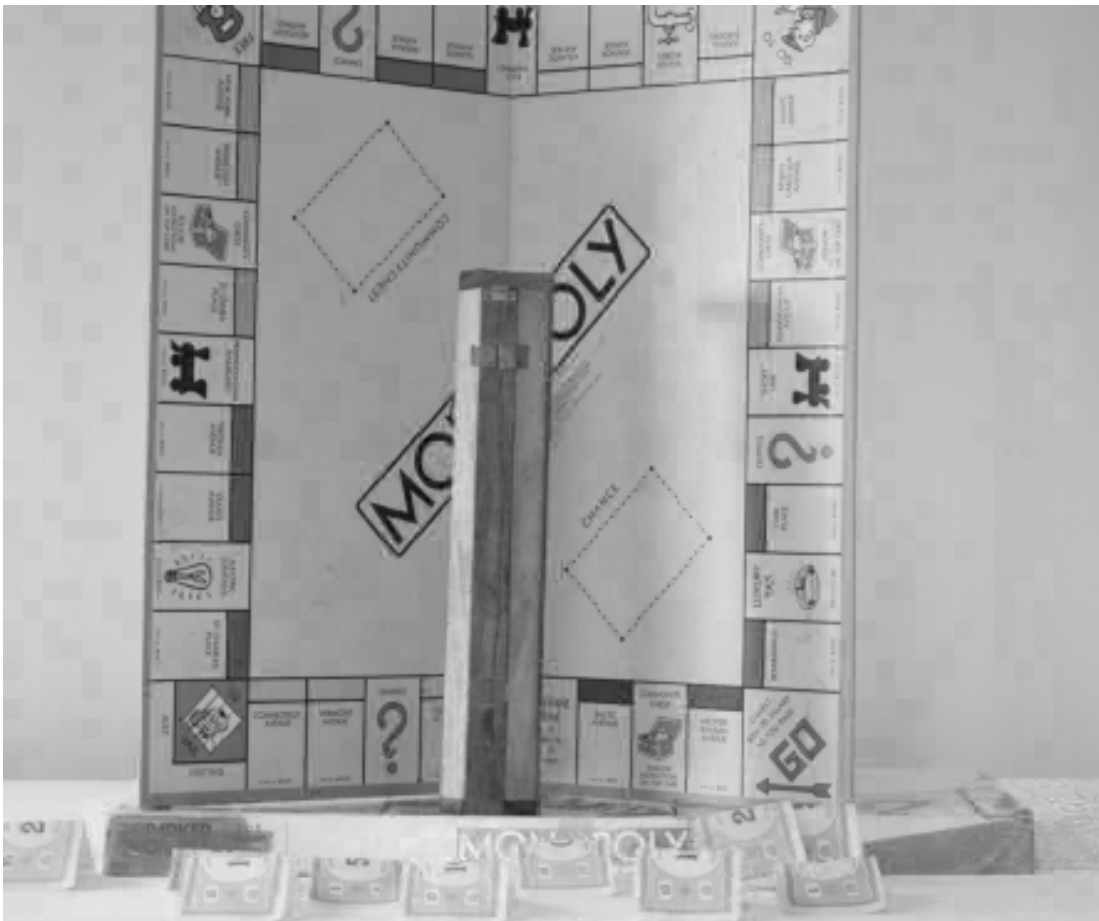


Figure 5.7: *Reconstructed right view with the FDCBM algorithm, ($PSNR = 35.48dB$, $SSIM = 0.95$, $b = 0.33bpp$).*



Figure 5.8: Reconstructed right view with the COMB algorithm, ($PSNR = 36.58dB$, $SSIM = 0.97$, $b = 0.33bpp$).

algorithm. Simulation results shown that it seems that this important difference between the two algorithms explain the interesting performances observed. An important study is investigated to understand, when and why, performances are significantly improved using some indicators based on spatial and disparity information.

Conclusion and Perspectives

Over the few last years, 3D imaging systems have been widely spread in numerous applications such as entertainment, industry, medical domains, cartography,... Stereoscopic technologies require the use of a pair of image representing the same scene from two different angles. This means that the amount of data needed for transmission and storage purposes is twice compared to that induced by a single image.

In order to exploit view redundancies, efficient compression techniques should be employed. The work carried out in this document falls within this context and this research work has been dedicated. To improve the Disparity Compensated Compression scheme, we are focused precisely on the disparity map estimation. This latter is very important because it impacts the performance in terms of rate distortion. In the following, we provide the main contributions we have made to this issue and mention some possible future perspectives.

The first contribution of this PhD thesis was the development of Disparity Compensated Block Matching (DCBM) algorithm showing that the proposed strategy gives better results in terms of rate distortion compared to the classical methods adopted by most image /video standards. In the proposed strategy, the blockwise disparity map is estimated taking into account the compensation effect. In the classical Disparity Compensated Compression (DCC) scheme, when using the Block Matching algorithm, a disparity map is estimated to predict the right image the most similar to its original version. Then the residual error, as the difference between the original view and its

prediction, is encoded yielding a refinement (i.e compensation) added to the predicted image. The DCBM algorithm operates differently. It improves first all the possible predicted images considering this refinement. Then the selected disparity map is the one with which the predicted view is most similar to that same original view. This shows a proof of the disparity map estimation concept controlled by the compensated view instead of the predicted one before compensation. Despite its efficiency compared to BM and R algorithms, the proposed DCBM algorithm has the drawback that it increases the numerical complexity. This is because of the necessity to code and decode a new residual error each time a new disparity value is considered. A possible way to make this algorithm faster is the including of parallel programming.

The second contribution of this dissertation was concerned to find a solution to the computational complexity problem of the DCBM algorithm. The proposed algorithm, called FDCBM (FastDCBM) algorithm, estimates the disparity map by a simplified modelling of how the coding of the residual error copes with the compensation. At this end, an analytic expression is derived based on the quantization of the DCT coefficients, cause of the distortion, in the JPEG coder. To select disparity, only 8×8 blocks of the residual error are used not the whole error with all its pixels. That is why the numerical complexity was significantly decreased compared to the previous DCBM-algorithm. The proposed FDCBM-algorithm performs better than the DCC using BM and R algorithms. This former could be integrated in the stereoscopic image/video coding to further improve the performance when using the DCC scheme.

The performance of the proposed DCBM and FDCBM algorithms can be improved in two ways: First, homologous blocks do not necessarily belongs to the same scan line. Hence disparities have an horizontal and vertical components. Second, search window can take real values. This gives more precision for the matching task.

The last contribution of this work focused on estimating disparity map given the best compromise between the minimization of the distortion of the predicted view and the bitrate allocated to encode the disparity map. The proposed algorithm, called COMB-algorithm, combines two algorithms: the R-algorithm performing better at low bitrates and the developed FDCBM one more efficient at high bitrates.

An important area of research will consist in the interpretation of the behaviour of the COMB algorithm at different bitrate ranges. Inspired from the TI (disparity information) and SI (spatial information) for characterizing the stereoscopic image as for video

sequence [58], it will be interesting to use this information to understand the relation between the nature of stereoscopic image and how our proposed COMB-algorithm behaves. Moreover, spatial and temporal information can be used with others features such as the variance, mean ,... of stereoscopic image as attribute for a technique of machine learning in order to categorize such image. Another perspective work may consider to extend the developed algorithms to stereoscopic videos. Last but not least, further investigations deserve to be conducted concerning the use the principle of our proposed methods with the motion compensation scheme.

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